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The supercurrent trace identities of the $N = 1$, $D = 4$ Super-Yang-Mills theory in the Wess-Zumino gauge

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Abstract: The supercurrent components of the $N = 1$, $D = 4$ Super-Yang-Mills theory in the Wess-Zumino gauge are coupled to the components of a background supergravitation field in the "new minimal" representation, in order to describe the various conservation laws in a functional way through the Ward identities for the diffeomorphisms and for the local supersymmetry, Lorentz and R-transformations. We also incorporate in the same functional formalism the supertrace identities, which leads however to a slight modification of the new minimal representation for supergravity, thus leading to a conformal version of it. The most general classical action obeying all the symmetry constraints and the condition of power-counting renormalizability is constructed.

Keywords: Gauge Symmetry, BRST Symmetry, Supergravity Models.

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1. Introduction

The renormalization properties of the supercurrent [1, 2] in $N = 1$ supersymmetric gauge theories (Super Yang-Mills – SYM) are of fundamental importance for the characterization of the ultraviolet behavior of the latter. Indeed, the supercurrent multiplet contains, together with the conserved spinor current associated to supersymmetry, the axial current associated to R-invariance and the *improved* conserved energy-momentum tensor. This implies that the anomaly of the axial current and the trace anomaly of the energy-momentum tensor – or scale anomaly, which controls the ultraviolet asymptotic behavior of the theory [3] – are related by supersymmetry. These anomalies appear as breakings of a set of "supertrace identities" including the usual energy-momentum trace identity, the spinor current trace identity and the divergence of the R axial current [1].

Thus, the use of the nonrenormalization theorem for the axial current [4, 5, 6] leads, through supersymmetry, to strong constraints on the scale anomaly, and even, in certain cases [7], to its absence[1].

The references [7] deal with this problem in the superfield formalism, which is well adapted if supersymmetry is exact[2]. But it is highly desirable to extend the discussion to the more realistic situation where supersymmetry is broken, possibly by mass terms. There, most of the advantages of superspace being lost, it is worthwhile to work in the component formalism, and better in the Wess-Zumino gauge where the number of component fields is minimal. As a preliminary step to this program, the renormalizability of the SYM theories in the Wess-Zumino gauge has been recently established in [10, 11, 12] for the unbroken case[3] and in [15] in the presence of "soft" breakings by mass terms.

The aim of the present paper is to set up a functional formalism describing, through Ward identities to be obeyed by the generating functional of the Green functions, both, the conservation laws and the supertrace identities for the components of the

[1] See [8] for an earlier application of this idea to a formal proof of the finiteness of $N = 4$ super-Yang-Mills.

[2] See [9] for a superfield approach, not relying on the supercurrent anomalies, and with application to broken supersymmetry.

[3] See [11, 13, 14] for extended supersymmetry $N = 2$ and $N = 4$.

supercurrent. We shall do it in the component formalism, in the Wess-Zumino gauge, proceeding in two steps.

The first step of our approach consists in the coupling of the supercurrent components to the components of a supermultiplet of supergravity fields considered as external (background) fields. The use of the so called "new minimal" realization of supergravity [16] allows to express all the conservation laws as the Ward identities for the local symmetries – diffeomorphisms, Lorentz, supersymmetry, R – of this model, which now is one of gauge and matter fields coupled to a classical supergravity background. We shall begin by reestablishing the associated BRS formulation [17] of these local symmetries [18] in our proper notations, taking into account the presence of the gauge and matter fields. We shall then check the off-shell nilpotency of the BRS operator after the introduction of the external fields coupled to those BRS transformations which are nonlinear in the dynamical fields - the so-called "Batalin-Vilkoviski antifields" [19] – and write down the Slavnov-Taylor identity, a nonlinear Ward identity containing all the information about the symmetries and their algebra, hence on the supercurrent conservation laws and the supercurrent algebra. The achievement of this program represents the supersymmetric generalization of the results found in [20] for the algebra of the energy-momentum tensor alone.

The second step is the incorporation of the supertrace identities into the formalism. As we shall see, in order to avoid hard breakings of some of the identities, we have to proceed to a slight reformulation of the coupling to the external supergravity fields, leading to a modified realization of the supergravity algebra with new symmetries corresponding to the supertrace identities. This actually represents a particular formulation of conformal supergravity coupled to matter.[4] We then give a full BRS formulation of this new algebra, with an off-shell nilpotent BRS operator and a new Slavnov-Taylor identity.

We construct the most general – power-counting renormalizable – action obeying the Slavnov-Taylor identity, and find in particular that the pure supergravitational part is the known action of conformal supergravity [21, 22].

We finally check the stability of the theory under small perturbations as a preliminary step towards the proof of the renormalizability [23]. Although the present paper deals exclusively with the classical theory, the stability proof needs only to be complemented by the study of the possible anomalies in order to prove the renormalizability of the theory and to characterize its ultraviolet properties.

The plan of the paper is as follows. In section 2 we introduce the coupling of matter and Yang-Mills supermultiplets with the supergravity multiplet in the "new minimal" formulation. (Super)trace identities and Weyl symmetry are discussed together with their breakings in section 3 within the same formalism, whereas the new formulation is introduced in section 4 where the full new BRS algebra is given, too. Section 5

[4] As far the authors know, this particular formulation of conformal supergravity did not yet appear in the literature.

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is devoted to the derivation of the most general action, including its pure conformal supergravity part, and to the proof of the stability of the theory under small perturbations.

Some technicalities are contained in the Appendices A to G.

2. $N = 1$ local SYM on curved space-time

As explained in the Introduction, we want to extend the previously studied model, $N = 1$ massless (global) SYM in the Wess-Zumino gauge [12], to the context of field theory on curved space-time. This means that we want to construct a model which is invariant under the *local* superpoincaré group rather than under the *global* superpoincaré group. But doing this is nothing but dealing with Supergravity coupled to SYM, with the crucial difference that we will consider the vierbein and the gravitino fields as *external* fields and doing so, we will not face the problem of dealing with a model which is power-counting non-renormalizable. In this way, we see that the title of this section could also have been "External $N = 1$ Supergravity coupled to SYM".

2.1. Field content of new minimal $N = 1$ supergravity

As one of the goal of this paper is to exhibit the link between the conservation law of the R-current and the trace of the energy-momentum tensor via the supertrace identity, we choose the *new* minimal formulation of supergravity [16] since it contains the source of the R-current, namely the field B_μ, as a gauge field. The vierbein and gravitino fields will be the sources of the energy-momentum tensor and of the supersymmetry current, respectively.

The field content of this formulation is [16]:

- the vierbein[5] $\mathrm{e}_\mu{}^a$,

- the gravitino $\Psi_{\mu\alpha}$,

- the gauge field B_μ of the R-gauge transformations,

- the gauge field $C_{\mu\nu}$ of the C-gauge transformations.

The ghost numbers, Grassmann parities, dimensions and R-weights of all the fields appearing in this paper are given in A. Our notations and conventions are given in B.

<u>Remark</u>: This is a torsion free formulation of gravity, which means that the Lorentz connection $w_{\mu ab}$ is a function of the vierbein (see C for a summary of the vierbein formalism).

The generalized BRS transformations [12, 24, 10, 11, 13, 14, 25], including diffeomorphisms, local Lorentz transformations, local supersymmetry and local R- and C-transformations are given by [16]

[5] See appendix C.

$$
\begin{aligned}
se_\mu{}^a &= \mathcal{L}_\xi e_\mu{}^a + \lambda^a{}_b e_\mu{}^b + 2\varepsilon\sigma^a\bar{\Psi}_\mu + 2\Psi_\mu\sigma^a\bar{\varepsilon} \\
s\Psi_\mu &= \mathcal{L}_\xi\Psi_\mu + \tfrac{i}{4}\lambda_{ab}\Psi_\mu\sigma^{ab} - i\eta\Psi_\mu - i\mathcal{D}_\mu\varepsilon \\
s\bar{\Psi}_\mu &= \mathcal{L}_\xi\bar{\Psi}_\mu - \tfrac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\Psi}_\mu + i\eta\bar{\Psi}_\mu - i\mathcal{D}_\mu\bar{\varepsilon} \\
sC_{\mu\nu} &= \mathcal{L}_\xi C_{\mu\nu} + \partial_\mu w_\nu - \partial_\nu w_\mu \\
&\qquad -2\varepsilon\sigma_\mu\bar{\Psi}_\nu - 2\Psi_\nu\sigma_\mu\bar{\varepsilon} + 2\varepsilon\sigma_\nu\bar{\Psi}_\mu + 2\Psi_\mu\sigma_\nu\bar{\varepsilon} \\
sB_\mu &= \mathcal{L}_\xi B_\mu + \partial_\mu\eta - \varepsilon\sigma_\mu\bar{\sigma}^{\nu\rho}\mathcal{D}_\nu\bar{\Psi}_\rho - \mathcal{D}_\nu\Psi_\rho\sigma^{\nu\rho}\sigma_\mu\bar{\varepsilon}\ ,
\end{aligned}
\tag{2.1}
$$

where $\xi^\mu, \lambda_{ab}, \varepsilon, \eta$ and w_μ are the ghosts for the diffeomorphisms, Lorentz transformations, supersymmetry, R- and C-transformations respectively – i.e., their infinitesimal parameters, but with opposite statistics. The σ^μ are the Pauli matrices and $\mathcal{L}_\xi$ is the Lie derivative.

Moreover, the following condensed notations have been used

$$
\begin{aligned}
\mathcal{D}_\mu\varepsilon &= \nabla_\mu\varepsilon + \tfrac{i}{4}F_{\mu ab}\varepsilon\sigma^{ab} + \tfrac{i}{4}V_\mu\varepsilon + \tfrac{1}{8}V^\lambda\varepsilon\sigma_{\mu\lambda} \\
\mathcal{D}_\mu\bar{\varepsilon} &= \nabla_\mu\bar{\varepsilon} - \tfrac{i}{4}F_{\mu ab}\bar{\sigma}^{ab}\bar{\varepsilon} - \tfrac{i}{4}V_\mu\bar{\varepsilon} + \tfrac{1}{8}V^\lambda\bar{\sigma}_{\mu\lambda}\bar{\varepsilon} \\
\mathcal{D}_\mu\Psi_\nu &= \nabla_\mu\Psi_\nu + \tfrac{i}{4}F_{\mu ab}\Psi_\nu\sigma^{ab} + \tfrac{i}{4}V_\mu\Psi_\nu + \tfrac{1}{8}V^\lambda\Psi_\nu\sigma_{\mu\lambda} \\
\mathcal{D}_\mu\bar{\Psi}_\nu &= \nabla_\mu\bar{\Psi}_\nu - \tfrac{i}{4}F_{\mu ab}\bar{\sigma}^{ab}\bar{\Psi}_\nu - \tfrac{i}{4}V_\mu\bar{\Psi}_\nu + \tfrac{1}{8}V^\lambda\bar{\sigma}_{\mu\lambda}\bar{\Psi}_\nu \\
F_{\mu ab} &= i\left(\Psi_a\sigma_b\bar{\Psi}_\mu - \Psi_b\sigma_a\bar{\Psi}_\mu + \Psi_a\sigma_\mu\bar{\Psi}_b - \Psi_b\sigma_\mu\bar{\Psi}_a + \Psi_\mu\sigma_a\bar{\Psi}_b - \Psi_\mu\sigma_b\bar{\Psi}_a\right) \\
V^\mu &= X^\mu + 4Y^\mu \\
X^\mu &= \varepsilon^{\mu\nu\rho\lambda}\partial_\nu C_{\rho\lambda} \\
Y^\mu &= i\varepsilon^{\mu\nu\rho\lambda}\Psi_\nu\sigma_\rho\bar{\Psi}_\lambda\ ,
\end{aligned}
\tag{2.2}
$$

where the symbol ∇_μ means covariant derivative with respect to usual gauge transformations (see subsection 2.2), diffeomorphisms, Lorentz transformations and R-transformations (see D for an explicit form of the covariant derivative).

Finally, the local algebra satisfied by these transformations [16] is encoded in the BRS transformations of the ghosts [18]:

$$
\begin{aligned}
s\xi^\mu &= \xi^\lambda\partial_\lambda\xi^\mu + 2iE^\mu \\
s\lambda_{ab} &= \mathcal{L}_\xi\lambda_{ab} + (\lambda^2)_{ab} + 2iE^\lambda\widetilde{w}_{\lambda ab} - \tfrac{i}{2}\varepsilon_{ab\mu\nu}E^\mu V^\nu \\
s\varepsilon &= \mathcal{L}_\xi\varepsilon + \tfrac{i}{4}\lambda_{ab}\varepsilon\sigma^{ab} - i\eta\varepsilon + 2E^\lambda\Psi_\lambda \\
s\bar\varepsilon &= \mathcal{L}_\xi\bar\varepsilon - \tfrac{i}{4}\lambda_{ab}\bar\sigma^{ab}\bar\varepsilon + i\eta\bar\varepsilon + 2E^\lambda\bar\Psi_\lambda \\
s\eta &= \mathcal{L}_\xi\eta - 2iE^\lambda B_\lambda \\
sw_\mu &= \mathcal{L}_\xi w_\mu + \partial_\mu w + 2iE_\mu + 2iC_{\mu\nu}E^\nu \\
sw &= \mathcal{L}_\xi w - 2iE^\lambda w_\lambda \ ,
\end{aligned}
\tag{2.3}
$$

where w is the ghost for the ghost w_μ and

$$
\begin{aligned}
E^\mu &= \varepsilon\sigma^\mu\bar\varepsilon \\
\widetilde{w}_{\mu ab} &= w_{\mu ab} + F_{\mu ab} \ .
\end{aligned}
\tag{2.4}
$$

Thanks to these transformation laws for the ghosts, this BRS operator s is nilpotent:

$$
s^2 = 0 \ . \tag{2.5}
$$

Remark: Let us note the presence in the ghost transformations of gauge field dependent terms, like $s\eta = \cdots - 2iE^\mu B_\mu$, which means that the anticommutator of two supersymmetries gives field dependent gauge transformations. This accounts for the fact that the supersymmetry algebra in the ordinary formulation – i.e. without ghosts – is not closed but *infinite*. This fact, already encountered in the rigid model [12], is typical of the Wess-Zumino gauge [26].

2.2. Field content of Super Yang-Mills and matter

The field content of the dynamical sector is [12, 15]:

- the gauge and gaugino fields A^i_μ and λ^i, in the adjoint representation of the gauge group $\mathcal{G}$, which is supposed to be simple,
- the scalar and fermion matter fields ϕ_A and ψ_A, in some (reducible) representation of $\mathcal{G}$ carried by the anti-Hermitian matrices T^i_{AB},
- the ghost, antighost and Lagrange multiplier fields c^i, $\bar c^i$ and b^i, in the adjoint representation.

The BRS transformations of these fields, including as well the usual BRS transformations associated with $\mathcal{G}$ (called $\mathcal{G}$-gauge transformations in this paper), are[6]

$$
\begin{aligned}
sA^i_\mu &= \mathcal{L}_\xi A^i_\mu + \nabla_\mu c^i + \varepsilon\sigma_\mu\bar{\lambda}^i + \lambda^i\sigma_\mu\bar{\varepsilon} \\
s\lambda^i &= \mathcal{L}_\xi\lambda^i - f^{ijk}c^j\lambda^k + \tfrac{i}{4}\lambda_{ab}\lambda^i\sigma^{ab} - i\eta\lambda^i - \tfrac{1}{2}\varepsilon\sigma^{\mu\nu}\widetilde{F}^i_{\mu\nu} - \tfrac{i}{2}G^2(\bar{\phi}_A T^i_{AB}\phi_B)\varepsilon \\
s\bar{\lambda}^i &= \mathcal{L}_\xi\bar{\lambda}^i - f^{ijk}c^j\bar{\lambda}^k - \tfrac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\lambda}^i + i\eta\bar{\lambda}^i + \tfrac{1}{2}\bar{\sigma}^{\mu\nu}\bar{\varepsilon}\widetilde{F}^i_{\mu\nu} + \tfrac{i}{2}G^2(\bar{\phi}_A T^i_{AB}\phi_B)\bar{\varepsilon} \\
s\phi_A &= \mathcal{L}_\xi\phi_A - T^i_{AB}c^i\phi_B - \tfrac{2}{3}i\eta\phi_A + 2\varepsilon\psi_A \\
s\bar{\phi}_A &= \mathcal{L}_\xi\bar{\phi}_A - T^i_{AB}c^i\bar{\phi}_B + \tfrac{2}{3}i\eta\bar{\phi}_A - 2\bar{\varepsilon}\bar{\psi}_A \\
s\psi_A &= \mathcal{L}_\xi\psi_A - T^i_{AB}c^i\psi_B + \tfrac{i}{4}\lambda_{ab}\psi_A\sigma^{ab} + \tfrac{1}{3}i\eta\psi_A + i\bar{\varepsilon}\bar{\sigma}^\mu\mathcal{D}_\mu\phi_A + 2\bar{\Lambda}_{(ABC)}\bar{\phi}_B\bar{\phi}_C\varepsilon \\
s\bar{\psi}_A &= \mathcal{L}_\xi\bar{\psi}_A - T^i_{AB}c^i\bar{\psi}_B - \tfrac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\psi}_A - \tfrac{1}{3}i\eta\bar{\psi}_A + i\bar{\sigma}^\mu\varepsilon\mathcal{D}_\mu\bar{\phi}_A - 2\Lambda_{(ABC)}\phi_B\phi_C\bar{\varepsilon} \\
sc^i &= \mathcal{L}_\xi c^i - \tfrac{1}{2}f^{ijk}c^jc^k - 2iE^\mu A^i_\mu \\
s\bar{c}^i &= \mathcal{L}_\xi\bar{c}^i + b^i \\
sb^i &= \mathcal{L}_\xi b^i - 2iE^\mu\partial_\mu\bar{c}^i \ ,
\end{aligned}
\tag{2.6}
$$

where G and $\Lambda_{(ABC)}$ are the gauge coupling constant and Yukawa coupling constants respectively[7]. Note that $\Lambda_{(ABC)}$ is totally symmetric in its indices. Moreover, the following notation has been used:

$$
\begin{aligned}
\widetilde{F}^i_{\mu\nu} &= F^i_{\mu\nu} - i\left\{\lambda^i\sigma_\mu\bar{\Psi}_\nu - \lambda^i\sigma_\nu\bar{\Psi}_\mu - \Psi_\nu\sigma_\mu\bar{\lambda}^i + \Psi_\mu\sigma_\nu\bar{\lambda}^i\right\} \\
\mathcal{D}_\mu\phi_A &= \nabla_\mu\phi_A - 2i\psi_A\Psi_\mu \\
\mathcal{D}_\mu\bar{\phi}_A &= \nabla_\mu\bar{\phi}_A + 2i\bar{\psi}_A\bar{\Psi}_\mu \ .
\end{aligned}
\tag{2.7}
$$

Due to the fact that the auxiliary fields of the gauge and matter multiplets have been eliminated, the nilpotency of s is true only *on-shell*, *i.e.*

$$
s^2 = \text{ equations of motion for } \lambda^i \text{ and } \psi_A \ . \tag{2.8}
$$

(Equations following from the action (2.12).)

For each field transforming nonlinearly in the dynamical fields we introduce an antifield which will couple to its s-variation [17, 19]. This happens for the dynamical fields themselves, except for $\bar{c}^i$ and b^i.

[6]The action of s on the gravitational fields are not modified by the adjunction of the usual BRS transformations associated with $\mathcal{G}$ since these fields are $\mathcal{G}$-gauge invariant.

[7]These couplings will appear in the action displayed only later on (See (2.12)), although the action cannot be dissociated from the formulation of the BRS transformations due to the auxiliary fields having been eliminated.

2.3. Flat limit

Before giving a BRS invariant action and the Slavnov-Taylor identity, let us have a look at the *flat limit* (F.L.) of the curved space-time model, defined as the limit where we recover the flat space-time and the rigid superpoincaré transformation group. On the gravitational fields, the flat limit is defined by

$$
\begin{aligned}
\mathrm{e}_\mu{}^a &\xrightarrow{\text{F.L.}} \delta_\mu^a \\
\Psi_\mu &\xrightarrow{\text{F.L.}} 0 \\
B_\mu &\xrightarrow{\text{F.L.}} 0 \\
C_{\mu\nu} &\xrightarrow{\text{F.L.}} 0 \ .
\end{aligned}
\tag{2.9}
$$

To be coherent we must specify the flat limit of the ghost in such a way that the BRS variation of the gravitational fields goes to zero as well, *i.e.*

$$
0 = s((\varphi)_{\text{F.L}}) \stackrel{!}{=} (s\varphi)_{\text{F.L}} \ , \tag{2.10}
$$

where $(\varphi)_{\text{F.L}}$ stands for the flat limit of φ. This is trivial for all the fields except for $\mathrm{e}_\mu{}^a$, which imposes the following flat limit for the ghosts:

$$
\begin{aligned}
\xi^\mu &\xrightarrow{\text{F.L.}} (\xi^\mu)_{\text{F.L}} - (\lambda^\mu{}_a)_{\text{F.L}} x^a \\
\lambda_{ab} &\xrightarrow{\text{F.L.}} (\lambda_{ab})_{\text{F.L}} \\
\varepsilon &\xrightarrow{\text{F.L.}} (\varepsilon)_{\text{F.L}} \\
\eta &\xrightarrow{\text{F.L.}} (\eta)_{\text{F.L}} \\
w_\mu &\xrightarrow{\text{F.L.}} 0 \\
w &\xrightarrow{\text{F.L.}} 0 \ ,
\end{aligned}
\tag{2.11}
$$

where $(\xi^\mu)_{\text{F.L}}, (\lambda_{ab})_{\text{F.L}}, (\varepsilon)_{\text{F.L}}, (\eta)_{\text{F.L}}$ are the constant ghosts used in the rigid model.

Remark: We see that when going from flat space-time to curved space-time, and thus from global Poincaré to local Poincaré transformations, the orbital part of the Lorentz transformation goes to the diffeomorphisms and only the spin part stays in the local Lorentz transformation.

2.4. Invariant action

An action which is invariant under s is given by

$$
\Sigma_{\text{inv}} = \Sigma_{\text{SYM}} + \Sigma_{\text{gf}} \ . \tag{2.12}
$$

In order to write down $\Sigma_{\rm SYM}$, let us introduce for convenience the following filtration operator:

$$N = \int d^4x \left\{ \Psi_\mu \frac{\delta}{\delta \Psi_\mu} - \bar{\Psi}_\mu \frac{\delta}{\delta \bar{\Psi}_\mu} + C_{\mu\nu} \frac{\delta}{\delta C_{\mu\nu}} \right\} , \tag{2.13}$$

which allows us to write

$$\Sigma_{\rm SYM} = \sum_{i=0}^{4} \Sigma^{(i)}_{\rm SYM} , \qquad \text{with} \qquad N \Sigma^{(i)}_{\rm SYM} = i \Sigma^{(i)}_{\rm SYM} . \tag{2.14}$$

Explicitly, we then have

$$\begin{aligned}
\Sigma^{(0)}_{\rm SYM} = \int d^4x \, \sqrt{g} \Big(& -\frac{1}{4G^2} F^{i\mu\nu} F^i_{\mu\nu} - \frac{i}{G^2} \lambda^i \sigma^\mu \nabla_\mu \bar{\lambda}^i - \frac{G^2}{8} |\bar{\phi}_A T^i_{AB} \phi_B|^2 + \frac{1}{2} \nabla_\mu \phi_A \nabla^\mu \bar{\phi}_A \\
& - i \psi_A \sigma^\mu \nabla_\mu \bar{\psi}_A - 2 \Lambda_{(ABC)} \phi_B \phi_C \bar{\Lambda}_{(ADE)} \bar{\phi}_D \bar{\phi}_E - i T^i_{AB} \phi_B \bar{\psi}_A \bar{\lambda}^i \\
& - i T^i_{AB} \bar{\phi}_A \psi_B \lambda^i + 2 \Lambda_{(ABC)} \psi_A \psi_B \phi_C + 2 \bar{\Lambda}_{(ABC)} \bar{\psi}_A \bar{\psi}_B \bar{\phi}_C - \frac{1}{12} R \phi_A \bar{\phi}_A \Big)
\end{aligned} \tag{2.15}$$

$$\begin{aligned}
\Sigma^{(1)}_{\rm SYM} = \int d^4x \, \sqrt{g} \Big(& - \frac{1}{2G^2} \left(\lambda^i \sigma^\mu \bar{\sigma}^{\nu\rho} \bar{\Psi}_\mu + \Psi_\mu \sigma^{\nu\rho} \sigma^\mu \bar{\lambda}^i \right) F^i_{\nu\rho} \\
& - \frac{i}{2} \left(\bar{\phi}_A T^i_{AB} \phi_B \right) \left(\Psi_\mu \sigma^\mu \bar{\lambda}^i + \lambda^i \sigma^\mu \bar{\Psi}_\mu \right) \\
& - i \left(\psi_A \sigma^\mu \bar{\sigma}^\nu \Psi_\mu \right) \nabla_\nu \bar{\phi}_A + i \left(\bar{\Psi}_\mu \bar{\sigma}^\nu \sigma^\mu \bar{\psi}_A \right) \nabla_\nu \phi_A \\
& - \frac{2}{3} \left(\bar{\psi}_A \bar{\sigma}^{\mu\nu} \nabla_\mu \bar{\Psi}_\nu \right) \phi_A - \frac{2}{3} \left(\nabla_\mu \Psi_\nu \sigma^{\mu\nu} \psi_A \right) \bar{\phi}_A \\
& + 2 \Lambda_{(ABC)} \left(\psi_A \sigma^\mu \bar{\Psi}_\mu \right) \phi_B \phi_C + 2 \bar{\Lambda}_{(ABC)} \left(\Psi_\mu \sigma^\mu \bar{\psi}_A \right) \bar{\phi}_B \bar{\phi}_C \Big)
\end{aligned} \tag{2.16}$$

$$\begin{aligned}
\Sigma^{(2)}_{\rm SYM} = \int d^4x \, \sqrt{g} \Big(& \frac{1}{G^2} \Big\{ -\frac{3}{8} V^\mu \left(\lambda^i \sigma_\mu \bar{\lambda}^i \right) + \left(\lambda^i \sigma^\lambda \bar{\Psi}_\nu \right) \left(\lambda^i \sigma^\nu \bar{\Psi}_\lambda \right) \\
& + \left(\Psi_\lambda \sigma^\nu \bar{\lambda}^i \right) \left(\Psi_\nu \sigma^\lambda \bar{\lambda}^i \right) + \frac{1}{2} \left(\lambda^i \sigma^\nu \bar{\Psi}_\nu \right) \left(\lambda^i \sigma^\lambda \bar{\Psi}_\lambda \right) \\
& + \frac{1}{2} \left(\Psi_\lambda \sigma^\lambda \bar{\lambda}^i \right) \left(\Psi_\nu \sigma^\nu \bar{\lambda}^i \right) + \frac{1}{2} \left(\lambda^i \sigma^\lambda \bar{\Psi}_\nu \right) \left(\Psi_\lambda \sigma^\nu \bar{\lambda}^i \right) \\
& + \frac{1}{2} \left(\lambda^i \sigma^\nu \bar{\Psi}_\nu \right) \left(\Psi_\lambda \sigma^\lambda \bar{\lambda}^i \right) - \left(\Psi_\lambda \sigma^\nu \bar{\lambda}^i \right) \left(\lambda^i \sigma_\nu \bar{\Psi}^\lambda \right) \Big\} \\
& + \frac{1}{8} X^\mu \left(\psi_A \sigma_\mu \bar{\psi}_A \right) + 2 \left(\psi_A \Psi_\mu \right) \left(\bar{\psi}_A \bar{\Psi}^\mu \right) \\
& + \frac{2i}{3} \bar{\Lambda}_{(ABC)} \bar{\phi}_A \bar{\phi}_B \bar{\phi}_C \left(\Psi_\mu \sigma^{\mu\nu} \Psi_\nu \right) + \frac{2i}{3} \Lambda_{(ABC)} \phi_A \phi_B \phi_C \left(\bar{\Psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\Psi}_\nu \right) \\
& + \frac{1}{6} \phi_A \bar{\phi}_A \nabla_\mu F_\nu{}^{\mu\nu} + \frac{i}{8} X^\mu \left(\phi_A \nabla_\mu \bar{\phi}_A - \nabla_\mu \phi_A \bar{\phi}_A \right) \\
& + \frac{1}{3} \phi_A \bar{\phi}_A \varepsilon^{\mu\nu\rho\lambda} \left(\Psi_\nu \sigma_\lambda \nabla_\mu \bar{\Psi}_\rho - \nabla_\mu \Psi_\nu \sigma_\lambda \bar{\Psi}_\rho \right) \Big)
\end{aligned} \tag{2.17}$$

$$\Sigma^{(3)}_{\rm SYM} = \int d^4x\,\sqrt{g}\Big(\phi_A\left(\bar\psi_A\bar\Psi_\mu\right)\left(-\frac{1}{4}V^\mu + \frac{1}{3}Y^\mu - \frac{i}{3}F_\nu{}^{\mu\nu}\right)$$
$$+\bar\phi_A\left(\psi_A\Psi_\mu\right)\left(-\frac{1}{4}V^\mu + \frac{1}{3}Y^\mu + \frac{i}{3}F_\nu{}^{\mu\nu}\right)\Big) \qquad (2.18)$$

$$\Sigma^{(4)}_{\rm SYM} = \int d^4x\,\sqrt{g}\Big(\frac{1}{32}\phi_A\bar\phi_A V_\mu V^\mu + \frac{1}{12}\phi_A\bar\phi_A\left\{F_\rho{}^{\mu\rho}F_{\nu\mu}{}^\nu + F^{\nu\mu\rho}F_{\mu\rho\nu}\right\}\Big)\ , \qquad (2.19)$$

and, for the gauge fixing part – of the Landau type:

$$\begin{aligned}\Sigma_{\rm gf} &= s\int d^4x\,\sqrt{g}\left\{-\partial_\mu\bar c^i g^{\mu\nu}A^i_\nu\right\}\\ &= \int d^4x\,\sqrt{g}\Big\{-2\left(\varepsilon\sigma^\lambda\bar\Psi_\lambda + \Psi_\lambda\sigma^\lambda\bar\varepsilon\right)\partial_\mu\bar c^i g^{\mu\nu}A^i_\nu - \partial_\mu b^i g^{\mu\nu}A^i_\nu\\ &\quad -2\partial_\mu\bar c^i\left(\varepsilon\sigma^\mu\bar\Psi^\nu + \varepsilon\sigma^\nu\bar\Psi^\mu + \Psi^\mu\sigma^\nu\bar\varepsilon + \Psi^\nu\sigma^\mu\bar\varepsilon\right)A^i_\nu\\ &\quad +\partial_\mu\bar c^i g^{\mu\nu}\left(\nabla_\nu c^i + \varepsilon\sigma_\nu\bar\lambda^i + \lambda^i\sigma_\nu\bar\varepsilon\right)\Big\}\ .\end{aligned} \qquad (2.20)$$

Remark: This action is the most general power-counting renormalizable[8] one invariant under s, however up to purely gravitational terms, such as the purely supergravity action, which we have omitted.

2.5. Classical identities defining the model

As usually in the framework of algebraic renormalization [6], we want to define the theory through a complete set of functional identities. These identities express the various symmetries of the model as well as specific properties linked to the choice made for the gauge fixing. The action (2.12) ought to be the general solution of these functional identities, up to field and parameter redefinitions.

2.5.1. The Slavnov-Taylor (ST) identity

To write down the ST identity – the Ward identity of BRS invariance – we add to the invariant action the following external action $\Sigma_{\rm ext}$ which contains, besides the usual antifields coupled to the nonlinear s-variations of the dynamical fields, also non-standard terms, quadratic in the antifields, in order to correct for the fact that the s-operator was on-shell rather than off-shell nilpotent:

$$\Sigma_{\rm ext} = \int d^4x\,\left(\sum_{\varphi\in I}\varphi^* s\varphi - \frac{G^2}{2\sqrt{g}}\left(\varepsilon\lambda^{*i} - \bar\varepsilon\bar\lambda^{*i}\right)^2 + \frac{2}{\sqrt{g}}\left(\varepsilon\psi^*_A\right)\left(\bar\varepsilon\bar\psi^*_A\right)\right)\ , \qquad (2.21)$$

[8]The most general invariant action, with the power-counting condition relaxed, may be found in [27].

where I is defined below, in (2.25). Then, for the total action Σ,

$$\Sigma = \Sigma_{\rm inv} + \Sigma_{\rm ext} \ , \tag{2.22}$$

we have the ST identity

$$\mathcal{S}(\Sigma) = 0 \ , \tag{2.23}$$

with

$$\mathcal{S}(\gamma) \equiv \int d^4x \left\{ \sum_{\varphi\in I} \frac{\delta\gamma}{\delta\varphi^*}\frac{\delta\gamma}{\delta\varphi} + \sum_{\varphi'\in I'} s\varphi' \frac{\delta\gamma}{\delta\varphi'} \right\} \ , \tag{2.24}$$

Here and throughout the paper, we define the three sets of fields

$$\begin{aligned} &I = \left\{ A^i_\mu, \lambda^i, \psi_A, \phi_A, c^i \right\} \ , \quad I^* = \left\{ A^{*i}_\mu, \lambda^{*i}, \psi^*_A, \phi^*_A, c^{*i} \right\} \ , \\ &I' = \{ \bar{c}^i, b^i \text{ and the supergravity fields and ghosts} \} \ . \end{aligned} \tag{2.25}$$

containing the dynamical fields which transform nonlinearly, the corresponding antifields and fields which transform linearly, including the gravitational fields and ghosts, respectively. For further use, we introduce the Batalin-Vilkovisky [19] notation for the ST operator:

$$\mathcal{S}(\gamma) = \frac{1}{2}(\gamma,\gamma) + b\gamma \ , \tag{2.26}$$

with

$$\begin{aligned} &(\gamma,\gamma') \equiv \int d^4x \left\{ \sum_{\varphi\in I} \left(\frac{\delta\gamma}{\delta\varphi^*}\frac{\delta\gamma'}{\delta\varphi} + \frac{\delta\gamma'}{\delta\varphi^*}\frac{\delta\gamma}{\delta\varphi} \right) \right\} \quad \forall\gamma,\gamma' \quad \text{with} \quad GP(\gamma) = GP(\gamma') = 0 \ , \\ &b \equiv \int d^4x \left\{ \sum_{\varphi'\in I'} s\varphi' \frac{\delta}{\delta\varphi'} \right\} \text{ with } \quad b^2 = 0 \ . \end{aligned} \tag{2.27}$$

From (2.23), we deduce that the linearized ST operator

$$\mathcal{B}_\Sigma = (\Sigma, \cdot) + b \ , \tag{2.28}$$

is nilpotent:

$$\mathcal{B}_\Sigma^2 = 0 \ . \tag{2.29}$$

2.5.2. Gauge condition and ghost identities

Besides the ST identity, we need to impose other identities:

The gauge condition

$$\frac{\delta\Sigma}{\delta b^i(x)} = \Omega^i_b(x) \ , \tag{2.30}$$

where $\Omega^i_b = \partial_\mu\left(\sqrt{g}g^{\mu\nu}A^i_\nu\right)$ is a classical breaking, *i.e.* is linear in the dynamical fields. It defines the Landau gauge in curved space-time.

The ghost identities

$$F^i\Sigma = \Delta^i_{\rm G} \ , \qquad \text{with} \qquad F^i = \int d^4x \left\{ \frac{\delta}{\delta c^i} - f^{ijk}\bar{c}^j \frac{\delta}{\delta b^k} \right\} \ . \tag{2.31}$$

This identity is peculiar to the Landau gauge [28] and $\Delta^i_{\rm G}$ is a classical breaking given in F. Moreover:

$$\begin{array}{lll} \dfrac{\delta\Sigma}{\delta\xi^\mu(x)} = \Omega_{{\rm D}\mu}(x) \ , & \dfrac{\delta\Sigma}{\delta\lambda_{ab}(x)} = \Omega^{ab}_{\rm L}(x) \ , & \dfrac{\delta\Sigma}{\delta\eta(x)} = \Omega_{\rm R}(x) \ , \\ \dfrac{\delta\Sigma}{\delta w_\mu(x)} = 0 \ , & \dfrac{\delta\Sigma}{\delta w(x)} = 0 \ , & \end{array} \tag{2.32}$$

where $\Omega_{{\rm D}\mu}, \Omega^{ab}_{\rm L}$ and $\Omega_{\rm R}$ are classical breakings given in F. The equations (2.32) follow from the diffeomorphisms, Lorentz transformations and R-transformations being linear and from the dynamical fields being invariant under the C-transformations.

By "commuting" the identities (2.30-2.32) with the ST identity (2.23), we recover the antighost equation, the $\mathcal{G}$-rigid invariance, and the Ward identities for the invariance under diffeomorphisms, Lorentz transformations, R- and C-transformations. They are explicitly given in F. This shows that all these symmetries are encoded in the ST identity, the gauge condition and the ghost identities, which constitute the minimal set of identities defining the model.

2.6. Supercurrent and associated Ward identities

Let us finish this section by explicitly giving the supercurrent components and conservation laws of the model.

Supercurrent components:

We identify the components of the supercurrent multiplet as the insertions

$$R^\mu = \frac{\delta\Sigma}{\delta B_\mu} \ , \quad Q^\mu{}_\alpha = \frac{\delta\Sigma}{\delta\Psi^\alpha_\mu} \ , \quad T^\mu{}_\nu = e_\nu{}^a \frac{\delta\Sigma}{\delta e_\mu{}^a} \ . \tag{2.33}$$

That they belong to a supermultiplet is encoded in the ε part – the proper supersymmetry part – of the transformation laws of the supergravity fields.

Conservation laws:

We readily see that the Ward identities for local R-invariance (last of (F.28), with (F.23)) and for the diffeomorphisms (third of (F.28), with (F.17)) correspond to the conservation of the current R^μ and of the energy-momentum tensor $T^\mu{}_\nu$:

$$\partial_\mu R^\mu = \sum_{\text{all fields}} iR_\varphi \varphi \frac{\delta\Sigma}{\delta\varphi} + \partial_\mu \left(\xi^\mu \frac{\delta\Sigma}{\delta\eta} \right) \ ,$$

$$
\partial_\mu T^\mu{}_\nu = \sum_{\text{all fields}} \partial_\nu\varphi \frac{\delta\Sigma}{\delta\varphi} + \partial_\mu\Bigg(\delta^\mu_\nu \sum_{\varphi^*\in I^*} \varphi^* \frac{\delta}{\delta\varphi^*} - A^i_\nu \frac{\delta}{\delta A^i_\mu} + A^{*i\mu}\frac{\delta}{\delta A^{*i\nu}} - \Psi_\nu \frac{\delta}{\delta\Psi_\mu} + \bar\Psi_\nu \frac{\delta}{\delta\bar\Psi_\mu}
$$

$$
-C_{\nu\rho}\frac{\delta}{\delta C_{\mu\rho}} - B_\nu \frac{\delta}{\delta B_\mu} - w_\nu \frac{\delta}{\delta w_\mu}\Bigg)\Sigma + \partial_\mu\left(\xi^\mu \frac{\delta\Sigma}{\delta\xi^\nu}\right) \ . \tag{2.34}
$$

The set I^* in the summation has been defined in (2.25).

Moreover, we can use the local Lorentz invariance (fourth of (F.28), with (F.21)) to show that this conserved energy-momentum tensor is symmetric. Indeed, its antisymmetric part vanishes on-shell:

$$
T^{[ab]} = \frac{i}{4}\lambda^i\sigma^{ab}\frac{\delta\Sigma}{\delta\lambda^i} - \frac{i}{4}\bar\lambda^i\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\lambda^i} + \frac{i}{4}\psi_A\sigma^{ab}\frac{\delta\Sigma}{\delta\psi_A} - \frac{i}{4}\bar\psi_A\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\psi_A} + \frac{i}{4}\Psi_\mu\sigma^{ab}\frac{\delta\Sigma}{\delta\Psi_\mu}
$$

$$
-\frac{i}{4}\bar\Psi_\mu\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\Psi_\mu} + \frac{i}{4}\varepsilon\sigma^{ab}\frac{\delta\Sigma}{\delta\varepsilon} - \frac{i}{4}\bar\varepsilon\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\varepsilon} + \left(\delta^a_c\lambda^b{}_d - \delta^b_c\lambda^a{}_d\right)\frac{\delta\Sigma}{\delta\lambda_{cd}}
$$

$$
+\partial_\mu\left(\frac{1}{2}\xi^\mu\frac{\delta\Sigma}{\delta\lambda_{ab}}\right) \ . \tag{2.35}
$$

As we will see later, $T^\mu{}_\nu$ is indeed the improved energy-momentum tensor, as its trace $T^\mu{}_\mu$ vanishes as well on-shell (4.20).

For the supersymmetry current $Q^\mu{}_\alpha$, and in the F.L., we use the local supersymmetry Ward identity (F.31), which yields

$$
\left(\partial_\mu Q^\mu{}_\alpha\right)\big|_{\text{ghosts}=0} = i\left(\Sigma, \frac{\delta\Sigma}{\delta\varepsilon^\alpha}\right)\Bigg|_{\text{ghosts}=0} + \text{ terms vanishing in the flat limit} \ . \tag{2.36}
$$

3. Weyl, supertrace and shift identities

In this section, we will exhibit other symmetries of the action Σ which are of particular interest for the purpose of our work. They are the Weyl symmetry [29, 22, 20], the so-called supertrace identities [1, 30, 2] which can now be written as a Ward identity thanks to the introduction of the external gravitational fields, and finally a new identity, called the shift identity, which links the $C_{\mu\nu}$ and B_μ dependence of the action. This last symmetry, as we will see, must be introduced in order to have a closed algebra.

3.1. Weyl identity

The Weyl transformations of infinitesimal *local* parameter $\kappa(x)$ are [29, 22, 20]

$$
\delta_\kappa\varphi(x) = \tilde d_\varphi \kappa(x)\varphi(x) \ , \tag{3.1}
$$

for all fields φ of the model. The Weyl weights $\tilde d_\varphi$ are given by

$$
\begin{aligned}
\tilde d_\varphi &= d_\varphi - \text{number of covariant indices } \mu \text{ of } \varphi_\mu \\
&\qquad + \text{number of contravariant indices } \mu \text{ of } \varphi^\mu, \qquad \text{for } \ \varphi\in I\cup I' \ , \\
\tilde d_{\varphi^*} &= -\tilde d_\varphi \ , \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } \ \varphi \in I \ ,
\end{aligned} \tag{3.2}
$$

the sets I and I' being defined by (2.25) and d_φ being the canonical dimension of φ. For further use, we write the associated Ward operator

$$S_{\rm Weyl} = \int d^4x \left\{ \sum_{\text{all fields}} \tilde{d}_\varphi \kappa \varphi \frac{\delta}{\delta\varphi} \right\} . \tag{3.3}$$

The Weyl transformations are in fact the generalization to the curved space-time formalism of the dilatation transformations [29, 22, 20].

By a direct calculation, we find

$$S_{\rm Weyl}\Sigma_{\rm SYM} = -\frac{3}{4}\int d^4x \left\{ \varepsilon_\mu{}^{\nu\rho\lambda} \partial_\nu \kappa C_{\rho\lambda} \frac{\delta \Sigma_{\rm SYM}}{\delta B_\mu} \right\} . \tag{3.4}$$

<u>Remark</u>: We see from (3.4) that although $\Sigma_{\rm SYM}$ is not invariant under *local* Weyl transformations, it is invariant under *global* Weyl transformations, as the r.h.s. of (3.4) vanishes for a constant infinitesimal parameter κ. This is directly linked to the dilatation invariance of the flat limit model.

Thanks to the operational form of the r.h.s of (3.4) we can define the following modified operator

$$S_\kappa = S_{\rm Weyl} + \frac{3}{4}\int d^4x \left\{ \varepsilon_\mu{}^{\nu\rho\lambda} \partial_\nu \kappa C_{\rho\lambda} \frac{\delta}{\delta B_\mu} \right\} , \tag{3.5}$$

to obtain

$$S_\kappa \Sigma_{\rm SYM} = 0 . \tag{3.6}$$

Next, applying S_κ on $\Sigma_{\rm ext}$ and $\Sigma_{\rm gf}$ leads to

$$\begin{aligned} S_\kappa \Sigma_{\rm ext} = \Delta_{\kappa,\rm ext} \equiv \int d^4x \Bigg(& - i\partial_\mu \kappa \left(\psi_A^* \sigma^\mu \bar\varepsilon \phi_A - \bar\psi_A^* \bar\sigma^\mu \varepsilon \bar\phi_A \right) \\ & + \frac{1}{2} \varepsilon_\mu{}^{\nu\rho\lambda} \partial_\nu \kappa C_{\rho\lambda} \left(\psi_A^* \sigma^\mu \bar\varepsilon \phi_A + \bar\psi_A^* \bar\sigma^\mu \varepsilon \bar\phi_A \right) \\ & + \partial_\mu \kappa \, \xi^\mu \left(\sum_{\varphi \in I} (-1)^{{\rm gh}(\varphi^*)} \, \tilde{d}_\varphi \varphi^* \varphi \right) \Bigg) , \end{aligned} \tag{3.7}$$

$$S_\kappa \Sigma_{\rm gf} = \Delta_{\kappa,\rm gf} \equiv \int d^4x \left(- 2\partial_\mu \kappa \left\{ \mathcal{B}_\Sigma \left(\sqrt{g} \bar{c}^i g^{\mu\nu} A_\nu^i \right) - \mathcal{L}_\xi \left(\sqrt{g} \bar{c}^i g^{\mu\nu} A_\nu^i \right) \right\} \right) , \tag{3.8}$$

and thus

$$S_\kappa \Sigma = \Delta_{\kappa,\rm ext} + \Delta_{\kappa,\rm gf} . \tag{3.9}$$

Again, we note that Σ, although not invariant under the local modified Weyl transformations, is invariant under the global Weyl transformations.

It should be stressed that, as it stands, (3.9) cannot be the starting point of the study of the quantum case since $\Delta_{\kappa,\rm gf}$ is nonlinear in the dynamical fields and thus needs to be correctly defined at the quantum level ($\Delta_{\kappa,\rm ext}$, being linear in the dynamical fields, is well defined in the quantum model). Although we are only interested in the classical

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model in this paper, it is only the first step towards a quantum treatment and thus a suitable form of (3.9) is needed. We will return to this problem later (see section (4.2)), where we will introduce sources in order to define the operator $\left(\sqrt{g}\bar{c}^i g^{\mu\nu} A^i_\nu\right)$ and its $\mathcal{B}_\Sigma$-variation at the quantum level.

Remark: As we see from the Ward operator (3.5), there was no need to introduce a proper gauge field associated to the local Weyl invariance. In a certain sense, it is B_μ – the gauge field of local R-invariance – which plays this role.

3.2. Supertrace identities

It is known [1, 30, 31, 32, 2] that the trace of the energy-momentum tensor, $T^\mu{}_\mu$, the trace of the supersymmetry current, $\sigma^\mu_{\alpha\dot\alpha}\bar{Q}^{\dot\alpha}_\mu$, and the divergence of the R-current, $\partial_\mu R^\mu$ – all vanishing on shell in the massless classical theory considered here – belong to a supermultiplet, and that the corresponding trace and conservation identities form a supermultiplet of equations: the "supertrace identities".

Our starting point will be the trace identity for the supersymmetry current, the other identities being deduced from it by covariance under supersymmetry. For the theory defined by the SYM action (2.14), it reads:

$$\sigma^\mu \frac{\delta\Sigma_{\rm SYM}}{\delta\bar\Psi_\mu} = 2\phi_A \frac{\delta\Sigma_{\rm SYM}}{\delta\psi_A} + 3\sigma^\nu\bar\sigma_\mu\Psi_\nu \frac{\delta\Sigma_{\rm SYM}}{\delta B_\mu} \ . \tag{3.10}$$

Introducing the infinitesimal local parameter $\chi_\alpha(x)$ – a Weyl spinor – we may rewrite this equation as an integral Ward identity:

$$\tilde{\tilde{S}}_\chi \Sigma_{\rm SYM} = 0 \ , \tag{3.11}$$

where we have defined the Ward operator $\tilde{\tilde{S}}_\chi$ by

$$\tilde{\tilde{S}}_\chi = \int d^4x \left\{ -\chi\sigma_\mu \frac{\delta}{\delta\bar\Psi_\mu} + 2\phi_A\chi\frac{\delta}{\delta\psi_A} + 3\left(\chi\sigma^\nu\bar\sigma_\mu\Psi_\nu\right)\frac{\delta}{\delta B_\mu} \right\} \ . \tag{3.12}$$

Applying $\tilde{\tilde{S}}_\chi$ to $\Sigma_{\rm ext}$ (2.21) leads to

$$\begin{aligned}
\tilde{\tilde{S}}_\chi \Sigma_{\rm ext} = & \int d^4x \Big(-\left(\lambda^{*i}\sigma^{\mu\nu}\varepsilon\right)\left(\chi\sigma_{\mu\nu}\lambda^i\right) + 4\left(\chi\varepsilon\right)\left(\bar\psi_A\bar\psi^*_A + \phi_A\phi^*_A\right) \\
& +4\bar\phi_A\left(\varepsilon\Psi_\mu\right)\left(\chi\sigma^\mu\bar\psi^*_A\right) - 4\phi_A\left(\chi\psi^*_A\right)\left(\Psi_\mu\sigma^\mu\bar\varepsilon\right) \\
& -2\phi_A\chi\partial_\mu\left(\xi^\mu\psi^*_A\right) - 2\left(\chi\psi^*_A\right)T^i_{AB}\phi_B c^i \\
& +\frac{i}{2}\phi_A\left(\chi\sigma^{ab}\psi^*_A\right)\lambda_{ab} + \frac{2i}{3}\phi_A\left(\chi\psi^*_A\right)\eta \Big) \\
= & \int d^4x \Big(2\left(\chi\psi^*_A\right)\frac{\delta\Sigma}{\delta\phi^*_A} + i\left(\chi\sigma_{ab}\varepsilon\right)\frac{\delta\Sigma}{\delta\lambda_{ab}} - 3i\left(\chi\varepsilon\right)\frac{\delta\Sigma}{\delta\eta} \\
& -2\left(\chi\varepsilon\right)\Big(\sum_{\varphi\in I}(-1)^{{\rm gh}(\varphi^*)}\,\tilde{d}_\varphi\varphi^*\varphi\Big)
\end{aligned}$$

$$+2i\phi_A\left(\chi\psi_A^*\right)\eta+\frac{i}{2}\phi_A\left(\chi\sigma^{ab}\psi_A^*\right)\lambda_{ab}-2\chi\partial_\mu\left(\xi^\mu\psi_A^*\phi_A\right)$$

$$+4\bar{\phi}_A\left(\varepsilon\Psi_\mu\right)\left(\chi\sigma^\mu\bar{\psi}_A^*\right)-4\phi_A\left(\chi\psi_A^*\right)\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\Bigg)\ , \tag{3.13}$$

Thus redefining $\tilde{\tilde{S}}_\chi$ by

$$\tilde{\tilde{S}}_\chi\to\tilde{S}_\chi=\int d^4x\,\Bigg\{-\chi\sigma_\mu\frac{\delta}{\delta\bar{\Psi}_\mu}+2\phi_A\chi\frac{\delta}{\delta\psi_A}+3\left(\chi\sigma^\nu\bar{\sigma}_\mu\Psi_\nu\right)\frac{\delta}{\delta B_\mu}$$
$$-2\left(\chi\psi_A^*\right)\frac{\delta}{\delta\phi_A^*}-i\left(\chi\sigma_{ab}\varepsilon\right)\frac{\delta}{\delta\lambda_{ab}}+3i\left(\chi\varepsilon\right)\frac{\delta}{\delta\eta}\Bigg\}\ , \tag{3.14}$$

and defining the classical breaking $\Delta_{\chi,\mathrm{ext}}$ by

$$\Delta_{\chi,\mathrm{ext}}=\int d^4x\,\Bigg(-2\left(\chi\varepsilon\right)\Bigg(\sum_{\varphi\in I}(-1)^{\mathrm{gh}(\varphi^*)}\ \tilde{d}_\varphi\varphi^*\varphi\Bigg)$$
$$+2i\phi_A\left(\chi\psi_A^*\right)\eta+\frac{i}{2}\phi_A\left(\chi\sigma^{ab}\psi_A^*\right)\lambda_{ab}-2\chi\partial_\mu\left(\xi^\mu\psi_A^*\phi_A\right)$$
$$+4\bar{\phi}_A\left(\varepsilon\Psi_\mu\right)\left(\chi\sigma^\mu\bar{\psi}_A^*\right)-4\phi_A\left(\chi\psi_A^*\right)\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\Bigg)\ , \tag{3.15}$$

allows to write

$$\tilde{S}_\chi\Sigma_{\mathrm{ext}}=\Delta_{\chi,\mathrm{ext}}\ . \tag{3.16}$$

Then, applying the modified operator on Σ_{gf} leads to

$$\tilde{S}_\chi\Sigma_{\mathrm{gf}}=\Delta_{\chi,\mathrm{gf}}+\int d^4x\left(4\left(\chi\varepsilon\right)\bar{c}^i\frac{\delta\Sigma_{\mathrm{gf}}}{\delta b^i}\right)\ ,$$
$$\text{with}\quad\Delta_{\chi,\mathrm{gf}}=\int d^4x\left(4\partial_\mu\left(\chi\varepsilon\right)\left(\sqrt{g}\bar{c}^ig^{\mu\nu}A_\nu^i\right)\right)\ . \tag{3.17}$$

We see again the appearance of the operator $\left(\sqrt{g}\bar{c}^ig^{\mu\nu}A_\nu^i\right)$ – quadratic in the dynamical fields – in the breaking of the supertrace identity, which deserves a better insight, postponed to section (4.2). We can redefine once more the operator $\tilde{S}_\chi$, absorbing the second term of the r.h.s. of (3.17) in it:

$$\tilde{S}_\chi\to S_\chi=\int d^4x\,\Bigg\{-\chi\sigma_\mu\frac{\delta}{\delta\bar{\Psi}_\mu}+3\left(\chi\sigma^\nu\bar{\sigma}_\mu\Psi_\nu\right)\frac{\delta}{\delta B_\mu}+2\phi_A\chi\frac{\delta}{\delta\psi_A}-2\left(\chi\psi_A^*\right)\frac{\delta}{\delta\phi_A^*}$$
$$-i\left(\chi\sigma_{ab}\varepsilon\right)\frac{\delta}{\delta\lambda_{ab}}+3i\left(\chi\varepsilon\right)\frac{\delta}{\delta\eta}-4\left(\chi\varepsilon\right)\bar{c}^i\frac{\delta}{\delta b^i}\Bigg\}\ . \tag{3.18}$$

Thus

$$S_\chi\Sigma=\Delta_{\chi,\mathrm{ext}}+\Delta_{\chi,\mathrm{gf}}\ . \tag{3.19}$$

For further use, let us denote by $\bar{S}_{\bar{\chi}}$ the complex conjugate of S_χ and by $\bar{\Delta}_{\bar{\chi},\mathrm{ext}}$ and $\bar{\Delta}_{\bar{\chi},\mathrm{gf}}$ the complex conjugates of $\Delta_{\chi,\mathrm{ext}}$ and $\Delta_{\chi,\mathrm{gf}}$, respectively.

Remark: The supertrace operator S_χ acts on the supergravity and matter fields, but not on the gauge fields.

3.3. Shift identity

Having written the new Ward identities (3.9) and (3.19) (and their complex conjugates), we have now to study the algebra they form with the identities defining the theory, and particularly with the ST identity (2.23). Thus, for any γ with zero GP, we have

$$\begin{aligned} S_\chi S(\gamma) - \mathcal{B}_\gamma \left(S_\chi \gamma - \Delta_{\chi,\mathrm{ext}} - \Delta_{\chi,\mathrm{gf}}\right) &= \left[S_\chi, b\right]\gamma + \left(\gamma, \Delta_{\chi,\mathrm{ext}}\right) + b\Delta_{\chi,\mathrm{ext}} + \mathcal{B}_\gamma \Delta_{\chi,\mathrm{gf}} \\ &= \dot{S}_\chi \gamma + b\Delta_{\chi,\mathrm{ext}} + \mathcal{B}_\gamma \Delta_{\chi,\mathrm{gf}} \ , \end{aligned} \tag{3.20}$$

where we have used the definitions (2.27,2.28), and with

$$\begin{aligned} \dot{S}_\chi &= S_\kappa\left[-2\left(\chi\varepsilon\right)\right] + S_K\left[-4\left(\chi\varepsilon\right)C_{\mu\nu} - 4i\left(\varepsilon\sigma_{\mu\nu}\chi\right)\right] \\ &\quad + S_\chi\left[\mathcal{L}_\xi\chi + \frac{i}{4}\lambda_{ab}\chi\sigma^{ab} + i\eta\chi - 2\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\chi\right] + \bar{S}_{\bar{\chi}}\left[-2\left(\varepsilon\Psi_\mu\right)\chi\sigma^\mu\right] \\ &\quad + \int d^4x \left\{-12i\left(\chi\sigma^\mu\bar{\varepsilon}\right)\left(\varepsilon\Psi_\mu\right)\frac{\delta}{\delta\eta} - 4\left(\chi\varepsilon\right)w_\mu\frac{\delta}{\delta w_\mu} - 4\left(\chi\varepsilon\right)w\frac{\delta}{\delta w}\right\} \ , \end{aligned} \tag{3.21}$$

where we have introduced the notation $S_\alpha[\ldots]$ for the Ward operator obtained by replacing the infinitesimal parameter $\alpha = \chi, \bar{\chi}, \kappa$ by the argument specified in the brackets, and have defined, for any antisymmetric tensor $K_{\mu\nu}$:

$$S_K = \int d^4x \left\{\frac{1}{2}K_{\mu\nu}\frac{\delta}{\delta C_{\mu\nu}} - \frac{3}{8}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu K_{\rho\lambda}\frac{\delta}{\delta B_\mu}\right\} \ . \tag{3.22}$$

Setting $\gamma = \Sigma$ in(3.20), we get

$$\dot{S}_\chi\Sigma + b\Delta_{\chi,\mathrm{ext}} + \mathcal{B}_\Sigma\Delta_{\chi,\mathrm{gf}} = 0 \ . \tag{3.23}$$

Next, using (3.21), (3.9), (3.19) and (2.32), we obtain the following (linearly broken) *shift identity*[9]

$$S_K\Sigma = \Delta_{K,\mathrm{ext}} \ , \tag{3.24}$$

where the classical breaking $\Delta_{K,\mathrm{ext}}$ is given by

$$\Delta_{K,\mathrm{ext}} = \int d^4x \left(-\frac{1}{4}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu K_{\rho\lambda}\left\{\psi_A^*\sigma^\mu\bar{\varepsilon}\phi_A + \bar{\psi}_A^*\bar{\sigma}^\mu\varepsilon\bar{\phi}_A\right\}\right) \ . \tag{3.25}$$

As the Ward operator S_K appears in the r.h.s of (3.20), its presence is necessary for closing the algebra.

Remark: The denomination of (3.24) as the shift identity can be justified by defining

$$\hat{B}_\mu = B_\mu + \frac{3}{8}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu C_{\rho\lambda} \ , \tag{3.26}$$

and thus getting, from (3.24)

$$\Sigma(B_\mu, C_{\mu\nu}, \varphi) = \Sigma(\hat{B}_\mu, 0, \varphi) + \left.\Delta_{K,\mathrm{ext}}\right|_{K_{\mu\nu}=C_{\mu\nu}} \ , \tag{3.27}$$

which shows that the action, except the "classical" term $\left.\Delta_{K,\mathrm{ext}}\right|_{K=C}$, depends on $C_{\mu\nu}$ only as a *shift* of B_μ.

[9] This identity can be directly checked on the action Σ given by (2.22).

3.4. Algebra

Let us finally show that the algebra formed by $S_\kappa, S_\chi, \bar{S}_{\bar\chi}, S_K$, the gauge condition (2.30), the ghost identities (2.31-2.32) and the ST identity (2.23) is closed, by exhibiting the nontrivial "commutation" rules. Thus, for any γ with zero GP, we have

$$\begin{aligned} S_K S(\gamma) - \mathcal{B}_\gamma \left(S_K\gamma - \Delta_{K,\mathrm{ext}}\right) &= \left[S_K, b\right]\gamma + \left(\gamma, \Delta_{K,\mathrm{ext}}\right) + b\Delta_{\chi,\mathrm{ext}} \\ &= \dot{S}_K\gamma + b\Delta_{\chi,\mathrm{ext}} \ , \end{aligned} \tag{3.28}$$

with

$$\begin{aligned} \dot{S}_K = {} & S_\chi\left[\frac{1}{8}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu K_{\rho\lambda}\bar\varepsilon\bar\sigma^\mu\right] + \bar{S}_{\bar\chi}\left[\frac{1}{8}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu K_{\rho\lambda}\varepsilon\sigma^\mu\right] \\ & + S_K\left[\mathcal{L}_\xi K_{\mu\nu}\right] + \int d^4x\,\left\{2iK_{\mu\nu}E^\nu\frac{\delta}{\delta w_\mu}\right\} \ . \end{aligned} \tag{3.29}$$

Similarly,

$$\begin{aligned} S_\kappa S(\gamma) - \mathcal{B}_\gamma\left(S_\kappa\gamma - \Delta_{\kappa,\mathrm{ext}} - \Delta_{\kappa,\mathrm{gf}}\right) &= \left[S_\kappa, b\right]\gamma + \left(\gamma, \Delta_{\kappa,\mathrm{ext}}\right) + b\Delta_{\chi,\mathrm{ext}} + \mathcal{B}_\gamma\Delta_{\kappa,\mathrm{gf}} \\ &= \dot{S}_\kappa\gamma + b\Delta_{\chi,\mathrm{ext}} + \mathcal{B}_\gamma\Delta_{\kappa,\mathrm{gf}} \ , \end{aligned} \tag{3.30}$$

with

$$\begin{aligned} \dot{S}_\kappa = {} & S_\chi\left[\left(\frac{i}{2}\partial_\mu\kappa - \frac{1}{4}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu\kappa C_{\rho\lambda}\right)\bar\varepsilon\bar\sigma^\mu\right] + \bar{S}_{\bar\chi}\left[\left(-\frac{i}{2}\partial_\mu\kappa - \frac{1}{4}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu\kappa C_{\rho\lambda}\right)\varepsilon\sigma^\mu\right] \\ & + S_K\left[-2\left(\partial_\mu w_\nu - \partial_\nu w_\mu\right)\right] + S_\kappa\left[\mathcal{L}_\xi\kappa\right] + \int d^4x\,\left\{-2\partial_\mu\kappa w\frac{\delta}{\delta w_\mu}\right\} \ . \end{aligned} \tag{3.31}$$

And finally,

$$\begin{aligned} & \left[S_\kappa, S_\chi\right] = S_\chi[-\frac{1}{2}\kappa\chi] \ , \ \left[S_\kappa, \bar{S}_{\bar\chi}\right] = \bar{S}_{\bar\chi}[-\tfrac{1}{2}\kappa\bar\chi] \ , \ \left[S_\kappa, S_K\right] = S_K[2\kappa K_{\mu\nu}] \ , \\ & \left[S_\kappa, \frac{\delta}{\delta b^i}\right] = -2\kappa\frac{\delta}{\delta b^i} \ , \ \left[S_\kappa, \frac{\delta}{\delta w_\mu}\right] = 2\kappa\frac{\delta}{\delta w_\mu} \ , \ \left[S_\kappa, \frac{\delta}{\delta w}\right] = 2\kappa\frac{\delta}{\delta w} \ , \end{aligned} \tag{3.32}$$

complete the list of the nontrivial commutation rules of the algebra.

4. New formulation of the model

We have thus obtained a formulation of the supertrace identities as a complete algebra of symmetries, extending the initial set of symmetries depicted in section 2 and forming a closed algebra. Although this result looks satisfactory for the classical theory, there is a feature which makes it unsuitable to a prompt generalization to the quantum case. This is the presence, among the breakings of the new symmetries, of the terms $\Delta_{\kappa,\mathrm{gf}}$ and $\Delta_{\chi,\mathrm{gf}}$ – originating from the gauge fixing part of the action – which are nonlinear in the dynamical fields (see (3.8) and (3.17)). Since these terms will suffer possible renormalizations, we have to formulate a new setup better adapted to the quantum extension of the model. This will be done by introducing a slightly modified symmetry content, but completely equivalent to the previous one at the classical level.

Thus, in the present section, we shall start again the construction of the model from the very beginning, including the new symmetries introduced in the last section in the BRS operator – the infinitesimal parameters κ, χ_α and $K_{\mu\nu}$ becoming ghosts with transformation laws assuring the nilpotency of the BRS operator. As we shall see, this together with the introduction of a doublet of external fields coupled to the integrands of the nonlinear breakings solves the "gauge fixing problem".

4.1. The C-gauge problem

With the introduction of the shift identity, we see that $C_{\mu\nu}$ looses its gauge field character since (3.24) means that *any* change of $C_{\mu\nu}$ (and not only the changes of the form $(\partial_\mu w_\nu - \partial_\nu w_\mu)$ which correspond to a C-gauge transformation) can be compensated by a suitable change of B_μ. This means that the ghost fields w_μ and w and the C-gauge transformations become useless as soon as we introduce the field $K_{\mu\nu}$ and the shift operator S_K (3.22).

Thus the first tremendous change we do in the model is at the level of the supergravitational field content, by skipping away the C-gauge invariance in favor of the shift identity. At the level of the BRS operator, care must be taken in order to preserve the nilpotency of s on $C_{\mu\nu}$, which is done by imposing the following BRS transformation for $K_{\mu\nu}$:

$$sK_{\mu\nu} = \mathcal{L}_\xi K_{\mu\nu} + 2i\partial_\mu \left(E_\nu + C_{\nu\rho}E^\rho\right) - 2i\partial_\nu \left(E_\mu + C_{\mu\rho}E^\rho\right) \ . \tag{4.1}$$

But an explicit calculation easily shows that s is not nilpotent on $K_{\mu\nu}$! To get the full nilpotency, we have to incorporate the new symmetries introduced in the previous section in the BRS formalism. We thus promote the fields $\kappa, \chi, \bar\chi$ and $K_{\mu\nu}$ to the rank of ghosts and add the Ward operators $S_\kappa, S_\chi, \bar S_{\bar\chi}$ and S_K to the BRS operator s we started with in (2.1), (2.3) and (2.6).

We then construct the BRS transformations of the new ghosts by inspecting the algebra displayed in the previous section. For example, the term

$$S_\chi \left[\mathcal{L}_\xi\chi + \frac{i}{4}\lambda_{ab}\chi\sigma^{ab} + i\eta\chi - 2\left(\Psi_\mu\sigma^\mu\bar\varepsilon\right)\chi\right] \tag{4.2}$$

in the r.h.s. of (3.21) means that the BRS transformation of χ looks like

$$s\chi = \mathcal{L}_\xi\chi + \frac{i}{4}\lambda_{ab}\chi\sigma^{ab} + i\eta\chi - 2\left(\Psi_\mu\sigma^\mu\bar\varepsilon\right)\chi + \cdots \ . \tag{4.3}$$

Doing this for the whole algebra displayed in (3.28) to (3.32) and checking for the

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nilpotency leads to the following BRS transformations for the new ghost fields:

$$
\begin{aligned}
sK_{\mu\nu} &= \mathcal{L}_\xi K_{\mu\nu} - 2\kappa K_{\mu\nu} + 2i\partial_\mu \left(E_\nu + C_{\nu\rho}E^\rho\right) - 2i\partial_\nu \left(E_\mu + C_{\mu\rho}E^\rho\right) \\
&\qquad +4i\left(\bar\chi\bar\sigma_{\mu\nu}\bar\varepsilon + \chi\sigma_{\mu\nu}\varepsilon\right) + 4\left(\bar\chi\bar\varepsilon + \chi\varepsilon\right)C_{\mu\nu} \\
s\chi &= \mathcal{L}_\xi\chi + \tfrac{i}{4}\lambda_{ab}\chi\sigma^{ab} + i\eta\chi + \tfrac{1}{2}\kappa\chi \\
&\qquad -2\left(\Psi_\mu\sigma^\mu\bar\varepsilon\right)\chi - 2\left(\varepsilon\bar\Psi_\mu\right)\bar\chi\bar\sigma^\mu - \left(\tfrac{1}{8}\left(M_\mu - 2N_\mu\right) + \tfrac{i}{2}\partial_\mu\kappa\right)\bar\varepsilon\bar\sigma^\mu \\
s\bar\chi &= \mathcal{L}_\xi\bar\chi - \tfrac{i}{4}\lambda_{ab}\bar\sigma^{ab}\bar\chi - i\eta\bar\chi + \tfrac{1}{2}\kappa\bar\chi \\
&\qquad -2\left(\varepsilon\sigma^\mu\bar\Psi_\mu\right)\bar\chi - 2\left(\varepsilon\Psi_\mu\right)\chi\sigma^\mu + \left(\tfrac{1}{8}\left(M_\mu - 2N_\mu\right) - \tfrac{i}{2}\partial_\mu\kappa\right)\varepsilon\sigma^\mu \\
s\kappa &= \mathcal{L}_\xi\kappa + 2\left(\bar\chi\bar\varepsilon + \chi\varepsilon\right) \ ,
\end{aligned}
\tag{4.4}
$$

where we have used the notations

$$
\begin{aligned}
M_\mu &= \varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu K_{\rho\lambda} \\
N_\mu &= \varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu \kappa C_{\rho\lambda} \ .
\end{aligned}
\tag{4.5}
$$

The action of the modified BRS operator s on the other fields can be found in E. With this generalized BRS operator, we still have, as in (2.8)

$$
s^2 = \text{ equations of motion for } \lambda^i \text{ and } \psi_A \ , \tag{4.6}
$$

where the equations of motion come from the invariant action (2.12).

4.2. Solution of the gauge fixing problem

In order to write a ST identity for the new formulation of the model, we have to solve the problem of the gauge fixing, namely we have to give a "suitable" definition of the two nonlinear insertions

$$
\begin{aligned}
\Omega^\mu &= -2\left(\sqrt{g}\bar c^i g^{\mu\nu} A^i_\nu\right) \\
\Lambda^\mu &= s\Omega^\mu - \mathcal{L}_\xi\Omega^\mu \ ,
\end{aligned}
\tag{4.7}
$$

which appear in the breakings of the supertrace identities (3.17) and of the Weyl symmetry (3.8). To do this, we introduce a doublet of sources I_μ and J_μ with the following BRS transformations:

$$
\begin{aligned}
sI_\mu &= \mathcal{L}_\xi I_\mu + J_\mu \\
sJ_\mu &= \mathcal{L}_\xi J_\mu - 2iE^\nu\partial_\nu I_\mu - 2i\partial_\mu E^\nu I_\nu \ ,
\end{aligned}
\tag{4.8}
$$

with $s^2 I_\mu = s^2 J_\mu = 0$. We then modify the action in

$$
\begin{aligned}
\Sigma \longrightarrow\ & \Sigma + s\int d^4x \left(I_\mu\Omega^\mu\right) \\
&= \Sigma + \int d^4x \left(J_\mu\Omega^\mu + I_\mu\Lambda^\mu\right) \ .
\end{aligned}
\tag{4.9}
$$

This allows the following operational definition (which is completely suitable to a quantum treatment of the model) for the two insertions Ω^μ and Λ^μ:

$$\begin{aligned}\Omega^\mu &= \frac{\delta\Sigma}{\delta J_\mu} \\ \Lambda^\mu &= \frac{\delta\Sigma}{\delta I_\mu} \ ,\end{aligned} \tag{4.10}$$

where we keep the notation Σ for the modified action.

4.3. Classical identities defining the new formulation of the model

Let us now rewrite the identities defining the theory, insisting on the changes made with respect to subsection 2.5.

The ST identity

The form of the ST identity has, of course, not changed, but the external action did since we modified the BRS operator s. So, we have

$$S(\Sigma) = 0 \ , \tag{4.11}$$

with

$$\Sigma = \Sigma_{\rm inv} + \Sigma_{\rm ext} \qquad \text{and} \qquad \Sigma_{\rm inv} = \Sigma_{\rm SYM} + \Sigma_{\rm gf} \ , \tag{4.12}$$

where $\Sigma_{\rm SYM}$ is given in (2.14-2.19), $\Sigma_{\rm ext}$ is as in (2.21) but with the BRS operator s modified by (4.4) and (4.8) (see E, equations (E.1-E.4) for the action of s on the other fields) and $\Sigma_{\rm gf}$ is the sum of the gauge fixing action of the old formulation (2.20) and of the modification done in (4.9). It can be written

$$\Sigma_{\rm gf} = s \int d^4x \, \sqrt{g} \, \{-\left(\partial_\mu + 2I_\mu\right) \bar{c}^i g^{\mu\nu} A^i_\nu\} \ . \tag{4.13}$$

The gauge condition

$$\frac{\delta\Sigma}{\delta b^i(x)} = \Omega^i_b(x) \ , \tag{4.14}$$

where $\Omega^i_b = (\partial_\mu - 2I_\mu)\left(\sqrt{g} g^{\mu\nu} A^i_\nu\right)$ is still a classical breaking, but is different from the one in (2.30).

The ghost identities

The following ghost identities are unchanged:

$$F^i\Sigma = \Delta^i_{\rm G} \ , \quad \frac{\delta\Sigma}{\delta\xi^\mu(x)} = \Omega_{{\rm D}\mu}(x) \ , \quad \frac{\delta\Sigma}{\delta\lambda_{ab}(x)} = \Omega^{ab}_{\rm L}(x) \ , \quad \frac{\delta\Sigma}{\delta\eta(x)} = \Omega_{\rm R}(x) \ . \tag{4.15}$$

Finally, the ghosts identities associated with the new fields are:

$$\frac{\delta\Sigma}{\delta K_{\mu\nu}(x)} = 0 \ , \quad \frac{\delta\Sigma}{\delta\chi(x)} = \Omega_\chi(x) \ , \quad \frac{\delta\Sigma}{\delta\bar\chi(x)} = \bar\Omega_{\bar\chi}(x) \ , \quad \mathcal{H}_\kappa(x)\Sigma = \Omega_\kappa(x) \ , \tag{4.16}$$

where the classical breakings $\Omega_\chi(x), \bar\Omega_{\bar\chi}(x)$ and $\Omega_\kappa(x)$ are given in F and where

$$\mathcal{H}_\kappa(x) = \frac{\delta}{\delta\kappa(x)} + \partial_\mu \frac{\delta}{\delta J_\mu(x)} \ . \tag{4.17}$$

Again, we can recover the different linear symmetries (now including the shift identity, the supertrace identities and the Weyl symmetry), by commuting the identities (4.14-4.16) with the ST identity (4.11). This is explicitly done in F.

4.4. Supercurrent Ward identities

Conservation laws:

We identify the components of the supercurrent multiplet of the new formulation in the same way as in the old formulation (see 2.33), and we use the Ward identities for local R-invariance, diffeomorphisms and supersymmetry displayed in F to get the conservation laws. We thus find that the conservation laws for the current R and for the supersymmetry are unchanged with respect to (2.34) and (2.36), whereas the conservation law for the energy-momentum tensor becomes

$$\begin{aligned}\partial_\mu T^\mu{}_\nu = & \sum_{\text{all fields}} \partial_\nu\varphi \frac{\delta\Sigma}{\delta\varphi} + \partial_\mu \left(\delta^\mu_\nu \sum_{\varphi^*\in I^*} \varphi^* \frac{\delta}{\delta\varphi^*} - A^i_\nu \frac{\delta}{\delta A^i_\mu} + A^{*i\mu}\frac{\delta}{\delta A^{*i\nu}} - \Psi_\nu \frac{\delta}{\delta\Psi_\mu} + \bar\Psi_\nu \frac{\delta}{\delta\bar\Psi_\mu} \right. \\ & \left. - C_{\nu\rho}\frac{\delta}{\delta C_{\mu\rho}} - B_\nu \frac{\delta}{\delta B_\mu} - K_{\nu\rho}\frac{\delta}{\delta K_{\mu\rho}} - I_\nu \frac{\delta}{\delta I_\mu} - J_\nu \frac{\delta}{\delta J_\mu} \right)\Sigma + \partial_\mu \left(\xi^\mu \frac{\delta\Sigma}{\delta\xi^\nu} \right) \ . \end{aligned} \tag{4.18}$$

Again, the local Lorentz invariance (fourth of (F.28), with (F.22)) shows that the energy-momentum tensor is symmetric:

$$\begin{aligned} T^{[ab]} = & \frac{i}{4}\lambda^i\sigma^{ab}\frac{\delta\Sigma}{\delta\lambda^i} - \frac{i}{4}\bar\lambda^i\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\lambda^i} + \frac{i}{4}\psi_A\sigma^{ab}\frac{\delta\Sigma}{\delta\psi_A} - \frac{i}{4}\bar\psi_A\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\psi_A} + \frac{i}{4}\Psi_\mu\sigma^{ab}\frac{\delta\Sigma}{\delta\Psi_\mu} \\ & - \frac{i}{4}\bar\Psi_\mu\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\Psi_\mu} + \frac{i}{4}\varepsilon\sigma^{ab}\frac{\delta\Sigma}{\delta\varepsilon} - \frac{i}{4}\bar\varepsilon\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\varepsilon} + \left(\delta^a_c\lambda^b{}_d - \delta^b_c\lambda^a{}_d \right)\frac{\delta\Sigma}{\delta\lambda_{cd}} \\ & + \frac{i}{4}\chi\sigma^{ab}\frac{\delta\Sigma}{\delta\chi} - \frac{i}{4}\bar\chi\sigma^{ab}\frac{\delta\Sigma}{\delta\bar\chi} + \partial_\mu\left(\frac{1}{2}\xi^\mu\frac{\delta\Sigma}{\delta\lambda_{ab}} \right) \ . \end{aligned} \tag{4.19}$$

Supertrace identities:

We proceed in the same way with the trace identities (second of (F.30), with (F.26) and last of (F.30), with (F.27)) corresponding to the supertrace identity and to the

Weyl identity, respectively, to get

$$
\begin{aligned}
\bar\sigma_\mu Q^\mu &= -2\bar\phi_a \frac{\delta\Sigma}{\delta\bar\psi_a} - 2\bar\psi^*_a \frac{\delta\Sigma}{\delta\bar\phi^*_a} + 3\bar\sigma^\nu \sigma_\mu \bar\Psi_\nu \frac{\delta\Sigma}{\delta B_\mu} + i\bar\sigma_{ab}\bar\varepsilon \frac{\delta\Sigma}{\delta\lambda_{ab}} + 4\bar\varepsilon\bar c^i \frac{\delta\Sigma}{\delta b^i} + 3i\bar\varepsilon \frac{\delta\Sigma}{\delta\eta} \\
&\quad - 2\bar\varepsilon \frac{\delta\Sigma}{\delta\kappa} + 2\left(\varepsilon\bar\Psi_\mu\right)\bar\sigma^\mu \frac{\delta\Sigma}{\delta\chi} + \partial_\mu\left(\xi^\mu \frac{\delta\Sigma}{\delta\bar\chi}\right) \\
&\quad + \left(-\frac{i}{4}\lambda_{ab}\bar\sigma^{ab} i\eta - \frac{1}{2}\kappa + 2\left(\varepsilon\sigma^\mu\bar\Psi_\mu\right)\right)\frac{\delta\Sigma}{\delta\bar\chi} \ , \\
T^\mu{}_\mu &= \sum_{\substack{\text{all fields} \\ \text{except } e_\mu{}^a}} \tilde d_\varphi \varphi \frac{\delta\Sigma}{\delta\varphi} - \frac{1}{4}\partial_\lambda\left\{\varepsilon_\rho{}^{\lambda\mu\nu} C_{\mu\nu}\left(3\frac{\delta\Sigma}{\delta B_\rho} + \bar\varepsilon\bar\sigma^\rho \frac{\delta\Sigma}{\delta\chi} + \varepsilon\sigma^\rho \frac{\delta\Sigma}{\delta\bar\chi}\right)\right\} \\
&\quad + \partial_\mu\left\{\frac{i}{2}\bar\varepsilon\bar\sigma^\mu \frac{\delta\Sigma}{\delta\chi} - \frac{i}{2}\varepsilon\sigma^\mu \frac{\delta\Sigma}{\delta\bar\chi} + \frac{\delta\Sigma}{\delta I_\mu} + \xi^\mu\left(\frac{\delta\Sigma}{\delta\kappa} + \partial_\nu \frac{\delta\Sigma}{\delta J_\nu}\right)\right\} \ .
\end{aligned}
\tag{4.20}
$$

The second of these identities shows that $T^\mu{}_\nu$ is indeed the improved energy-momentum tensor, being on-shell traceless.

Finally, we explicitly give the shift identity (first of (F.30), with (F.25))

$$
\frac{\delta\Sigma}{\delta C_{\mu\nu}} = -\frac{1}{4}\partial_\lambda\left\{\varepsilon_\rho{}^{\lambda\mu\nu} C_{\mu\nu}\left(3i\frac{\delta\Sigma}{\delta B_\rho} + \bar\varepsilon\bar\sigma^\rho \frac{\delta\Sigma}{\delta\chi} + \varepsilon\sigma^\rho \frac{\delta\Sigma}{\delta\bar\chi}\right)\right\} \ , \tag{4.21}
$$

which shows that the insertion $\dfrac{\delta\Sigma}{\delta C_{\mu\nu}}$ is not independent.

4.5. Flat limit

Defining the F.L. of the new formulation of the model will allow to recognize in more familiar terms the improvements implied by the adjunction of the new symmetries. First, to keep track of the supertrace identity and of the Weyl transformations when going to the F.L., we define

$$
\begin{aligned}
\chi &\xrightarrow{\text{F.L.}} (\chi)_{\text{F.L}} \\
\kappa &\xrightarrow{\text{F.L.}} (\kappa)_{\text{F.L}} \ .
\end{aligned}
\tag{4.22}
$$

Next we study the constraint of coherence (2.10), which leads to

$$
\begin{aligned}
K_{\mu\nu} &\xrightarrow{\text{F.L.}} 0 \\
\varepsilon \quad &\xrightarrow{\text{F.L.}} (\varepsilon)_{\text{F.L}} - ix^\mu(\bar\chi\bar\sigma_\mu)_{\text{F.L}} \ ,
\end{aligned}
\tag{4.23}
$$

the first one coming from imposing $(sC_{\mu\nu})_{\text{F.L}} = 0$ and the second from imposing $(s\Psi_\mu)_{\text{F.L}} = 0$, since both $C_{\mu\nu}$ and Ψ_μ have a null F.L. (See section 2.3).

Applying then (2.10) to κ leads to a new problem: the r.h.s tends to an x-dependent term (coming from the x-dependent term of the flat limit of ε) whereas the l.h.s is constant. This problem is solved by introducing a new constant ghost $(\beta_\mu)_{\text{F.L}}$ and to redefine the F.L. of κ as

$$
\kappa \xrightarrow{\text{F.L.}} (\kappa)_{\text{F.L}} + x^\mu(\beta_\mu)_{\text{F.L}} \ . \tag{4.24}
$$

Finally applying (2.10) to the remaining ghosts leads to

$$
\begin{aligned}
\xi^\mu \quad &\xrightarrow{\text{F.L.}} (\xi^\mu)_{\text{F.L}} - (\lambda^\mu{}_a)_{\text{F.L}} x^a + (\kappa)_{\text{F.L}} x^\mu - \tfrac{1}{2} x^2 (\beta^\mu)_{\text{F.L}} + x^\mu x^\nu (\beta_\nu)_{\text{F.L}} \\
\lambda_{ab} \quad &\xrightarrow{\text{F.L.}} (\lambda_{ab})_{\text{F.L}} + (\beta_a)_{\text{F.L}} x_b - (\beta_b)_{\text{F.L}} x_a \\
\varepsilon \quad &\xrightarrow{\text{F.L.}} (\varepsilon)_{\text{F.L}} - i x^\mu (\bar\chi \bar\sigma_\mu)_{\text{F.L}} \\
\eta \quad &\xrightarrow{\text{F.L.}} (\eta)_{\text{F.L}} \\
K_{\mu\nu} &\xrightarrow{\text{F.L.}} 0 \\
\chi \quad &\xrightarrow{\text{F.L.}} (\chi)_{\text{F.L}} \\
\kappa \quad &\xrightarrow{\text{F.L.}} (\kappa)_{\text{F.L}} + x^\mu (\beta_\mu)_{\text{F.L}} \ .
\end{aligned}
\tag{4.25}
$$

Looking at the F.L. of the transformations laws of the dynamical fields and at the algebra encoded in (4.25) clearly shows that we have thus recovered the whole superconformal algebra [22, 2], where the ghosts $(\kappa)_{\text{F.L}}, (\beta_\mu)_{\text{F.L}}$ and $(\chi)_{\text{F.L}}$ are the ghosts for the dilatations, the special conformal transformations and the special superconformal transformations respectively.

5. Stability of the solution

As a first step to a complete quantum treatment of the model, let us study the stability of the classical action Σ (4.12) under radiative corrections. That means looking for the most general classical integrated insertion Σ_c (having the same quantum numbers and dimension as Σ) such that the perturbed action

$$\Sigma' = \Sigma + \zeta \Sigma_c \ , \tag{5.1}$$

satisfies, at the first order in the infinitesimal parameter ζ, the same constraints as Σ, *i.e.* the ST identity (4.11), the gauge condition (4.14) and the ghost identities (4.15) and (4.16). The action is said to be stable if Σ_c corresponds to genuine counterterms, *i.e.* if Σ_c can be reabsorbed through a redefinition of the field amplitudes and of the parameters (namely the coupling constants).

Expanding these constraints on Σ' at the first order in ζ leads to the following set of equations that must be satisfied by Σ_c:

$$\mathcal{B}_\Sigma \Sigma_c = 0 \ , \tag{5.2}$$

$$
\begin{aligned}
&\frac{\delta \Sigma_c}{\delta b^i} = 0 \ , \quad F^i \Sigma_c = 0 \ , \\
&\frac{\delta \Sigma_c}{\delta \xi^\mu} = \frac{\delta \Sigma_c}{\delta \lambda_{ab}} = \frac{\delta \Sigma_c}{\delta \eta} = 0 \ , \\
&\frac{\delta \Sigma_c}{\delta K_{\mu\nu}} = \frac{\delta \Sigma_c}{\delta \chi} = \frac{\delta \Sigma_c}{\delta \bar\chi} = \mathcal{H}_\kappa \Sigma_c = 0 \ .
\end{aligned}
\tag{5.3}
$$

The constraints (5.3) are easily solved and lead to a Σ_c which is independent of $b^i, \xi^\mu, \lambda_{ab}, \eta, K_{\mu\nu}, \chi, \bar\chi$ and which depends on c^i only through its derivative $\partial_\mu c^i$ and on κ only through the combination

$$\hat J_\mu = J_\mu + \partial_\mu \kappa \ . \tag{5.4}$$

The equations (5.2) constitutes a cohomology problem, due to the nilpotency of the linearized ST operator $\mathcal{B}_\Sigma$ (see (2.29)). The strategy we applied to construct the explicit solution is given in G and we give here the result in the form:

$$\Sigma_c = \Sigma_{\rm ph} + \mathcal{B}_\Sigma \hat\Sigma_c \ , \tag{5.5}$$

with

$$\Sigma_{\rm ph} = Z_G \frac{\partial\Sigma}{\partial G} + Z_{(ABC)} \frac{\partial\Sigma}{\partial\Lambda_{(ABC)}} + \bar Z_{(ABC)} \frac{\partial\Sigma}{\partial\bar\Lambda_{(ABC)}} + \Sigma_{\rm CSG} \ , \tag{5.6}$$

and

$$\begin{aligned}\hat\Sigma_c = \int d^4x \Big(& Z_A A^i_\mu \hat A^{*i\mu} + Z_\lambda \lambda^{*i}\lambda^i + \bar Z_{\bar\lambda} \bar\lambda^{*i}\bar\lambda^i \\ & + Z_{\phi AB} \left(\phi^*_A + \frac{1}{2}\bar\Psi_\mu \bar\sigma^\mu \psi^*_A\right)\phi_B + \bar Z_{\phi AB}\left(\bar\phi^*_A - \frac{1}{2}\Psi_\mu\sigma^\mu\bar\psi^*_A\right)\bar\phi_B \\ & + Z_{\psi AB}\psi^*_A\left(\psi_B + \frac{1}{2}\sigma^\mu\bar\Psi_\mu\phi_B\right) + \bar Z_{\psi AB}\bar\psi^*_A\left(\bar\psi_B - \frac{1}{2}\bar\sigma^\mu\Psi_\mu\bar\phi_B\right) \\ & + V g^{\mu\rho}\left(\Psi_\mu + \frac{i}{3}\Psi_\nu\sigma^\nu{}_\mu\right)\lambda^{*i}A^i_\rho + \bar V g^{\mu\rho}\left(\bar\Psi_\mu + \frac{i}{3}\bar\Psi_\nu\bar\sigma^\nu{}_\mu\right)\bar\lambda^{*i}A^i_\rho \Big) \ . \end{aligned} \tag{5.7}$$

The constant Z_G and the $\mathcal{G}$-invariant tensor $Z_{(ABC)}$ are interpreted as renormalizations of the coupling constants G and $\Lambda_{(ABC)}$ respectively, whereas the constants Z_A, Z_λ, V and the $\mathcal{G}$-invariant tensors $Z_{\phi AB}, Z_{\psi AB}$ correspond to unphysical field renormalizations. Finally, the counterterm $\Sigma_{\rm CSG}$ – which is independent of the dynamical fields – is in fact the conformal supergravity action [21, 22], but is interpreted in our context of external supergravity as corrections to the current algebra. The complete form of $\Sigma_{\rm CSG}$ can be found in [21, 22], and it starts as

$$\Sigma_{\rm CSG} = Z_{\rm CSG} \int d^4x \sqrt{g}\left(C^{\mu\nu\rho\lambda}C_{\mu\nu\rho\lambda} - \frac{3}{4}\hat G^{\mu\nu}\hat G_{\mu\nu} + \cdots\right) \ , \tag{5.8}$$

where the Weyl tensor $C^{\mu\nu\rho\lambda}$ and the R-curvature tensor $\hat G_{\mu\nu}$ are defined in G, equations (G.43) and (G.44).

Remark: At the F.L. the V-term and $\Sigma_{\rm CSG}$ disappear and the other ones tend, as expected, to the counterterms found in the rigid model [12].

6. Conclusion

We have thus succeeded in constructing a general functional formalism which incorporates, within a unique Slavnov-Taylor identity associated to a general BRS invariance, both the set of conservation laws of the supercurrent components, and the set of the "supertrace identities". The latter includes the – potentially anomalous – Ward identities for the traces of the energy-momentum tensor and spinor current (4.20), as well as the shift identity (4.21). In order to achieve this, we had to reformulate the "new minimal" external supergravity in such a way that the conformal properties implied by the supertrace identities might be naturally incorporated.

The supermultiplet structure of the supertrace identities – which is obvious in the superspace approach, where they form a superfield – is here somewhat hidden in the algebra of the Slavnov-Taylor operator. One can nevertheless observe it in the algebraic identities (3.20),(3.28),(3.30) and (3.32). We have given the algebra explicitly for the old formulation. But, in the new formulation, designed indeed in order to take care of it, the complete algebra can be read out directly from the BRS transformations of the ghosts (see (4.4)).

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A. Quantum numbers of the fields

We give in this Appendix the ghost numbers, Grassmann parities, dimensions and R-weights of all the fields appearing in the model.

They commute or anticommute according to the formula:

$$\varphi_1\varphi_2 = (-1)^{\mathrm{GP}(\varphi_1)\cdot\mathrm{GP}(\varphi_2)}\varphi_2\varphi_1 \ . \tag{A.1}$$

Table 1 on page 27 shows the Grassmann parities GP, the ghost numbers $\Phi\Pi$, the dimensions d and the R-weights of the various fields.

B. Notations and conventions

Units: $\hbar = c = 1$.

Flat space-time metric: $(\eta_{ab}) = \mathrm{diag}(1,-1,-1,-1)$, $(a,b = 0,1,2,3)$.

Weyl spinor: $(\psi_\alpha \ , \ \alpha = 1,2) \ \in$ repr. $(\frac{1}{2},0)$ of the Lorentz group. $\psi_\alpha\psi'_\beta = -\psi'_\beta\psi_\alpha$

Complex conjugate spinor: $(\bar\psi_{\dot\alpha} \ , \ \dot\alpha = 1,2) \ \in$ repr. $(0,\frac{1}{2})$.

	A^i_μ	λ^i	ϕ_A	ψ_A	c^i	$\bar{c}^i$	b^i	$A^{*i\mu}$	λ^{*i}	ϕ^*_A	ψ^*_A	c^{*i}	$\mathrm{e}_a{}^\mu$	Ψ_μ	B_μ	$C_{\mu\nu}$
GP	0	1	0	1	1	1	0	1	0	1	0	0	0	1	0	0
$\Phi\Pi$	0	0	0	0	1	-1	0	-1	-1	-1	-1	-2	0	0	0	0
d	1	3/2	1	3/2	0	2	2	3	5/2	3	5/2	4	0	1/2	1	0
R	0	-1	-2/3	1/3	0	0	0	0	1	2/3	-1/3	0	0	-1	0	0

	ξ^μ	λ_{ab}	ε	η	w_μ	w	$K_{\mu\nu}$	χ	κ	I_μ	J_μ
GP	1	1	0	1	1	0	1	0	1	0	1
$\Phi\Pi$	1	1	1	1	1	2	1	1	1	0	1
d	-1	0	-1/2	0	-1	-2	0	1/2	0	1	1
R	0	0	-1	0	0	0	0	1	0	0	0

Table 1: Grassmann parity GP, ghost number $\Phi\Pi$, dimensions d, and R-weight of the dynamical fields and their antifields.

Raising and lowering of spinor indices: (the same for dotted indices)

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta \ , \quad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \ , \tag{B.1}$$

with

$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}, \quad \varepsilon^{12} = 1, \quad \varepsilon_{\alpha\beta} = -\varepsilon^{\alpha\beta}, \quad \varepsilon^{\alpha\beta}\varepsilon_{\beta\gamma} = \delta^\alpha_\gamma \, . \tag{B.2}$$

Derivative with respect to a spinor component: (the same for dotted indices)

$$\frac{\partial}{\partial\psi^\alpha}\psi^\beta = \delta^\beta_\alpha \ , \quad \frac{\partial}{\partial\psi_\alpha} = \varepsilon^{\alpha\beta}\frac{\partial}{\partial\psi^\beta} \ , \tag{B.3}$$

Pauli matrices:

$$\left(\sigma^a_{\alpha\dot\beta}\right) = \left(\sigma^0_{\alpha\dot\beta},\, \sigma^1_{\alpha\dot\beta},\, \sigma^2_{\alpha\dot\beta},\, \sigma^3_{\alpha\dot\beta}\right) \ , \qquad \bar\sigma_a^{\dot\alpha\beta} = \sigma_a^{\beta\dot\alpha} = \varepsilon^{\beta\alpha}\varepsilon^{\dot\alpha\dot\beta}\sigma_{a\,\alpha\dot\beta} \ , \tag{B.4}$$

$$(\sigma^{ab})_\alpha{}^\beta = \frac{i}{2}\Big[\sigma^a\bar\sigma^b - \sigma^b\bar\sigma^a\Big]_\alpha{}^\beta \ , \qquad (\bar\sigma^{ab})^{\dot\alpha}{}_{\dot\beta} = \frac{i}{2}\Big[\bar\sigma^a\sigma^b - \bar\sigma^b\sigma^a\Big]^{\dot\alpha}{}_{\dot\beta} \ , \tag{B.5}$$

with

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ , \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ ,$$

$$\bar\sigma^0 = \sigma^0 \ , \quad \bar\sigma^i = -\sigma^i = \sigma_i \ , \quad \sigma^{0i} = -\bar\sigma^{0i} = -i\sigma^i \ , \quad \sigma^{ij} = \bar\sigma^{ij} = \varepsilon^{ijk}\sigma^k \ ,$$

$$i, j, k = 1, 2, 3 \ .$$

<u>Remark</u>: We define $\sigma_\mu \equiv \mathrm{e}_\mu{}^a\sigma_a$. (See appendix C for the definition of the vierbein $\mathrm{e}_\mu{}^a$.)

Summation conventions and complex conjugation: Let ψ and χ be two Weyl (anticommutant) spinors; we have:

$$\psi\chi = \psi^\alpha\chi_\alpha = -\chi_\alpha\psi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \ ,$$
$$\bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = -\bar\chi^{\dot\alpha}\bar\psi_{\dot\alpha} = \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = \bar\chi\bar\psi \ ,$$
$$\psi\sigma^\mu\bar\chi = \psi^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\chi^{\dot\alpha} \ , \quad \bar\psi\bar\sigma_\mu\chi = \bar\psi_{\dot\alpha}\bar\sigma_\mu^{\dot\alpha\alpha}\chi_\alpha \ ,$$
$$\overline{(\psi\chi)} = \bar\chi\bar\psi = \bar\psi\bar\chi \ ,$$
$$\overline{(\psi\sigma^\mu\bar\chi)} = \chi\sigma^\mu\bar\psi = -\bar\psi\bar\sigma^\mu\chi \ ,$$
$$\overline{(\psi\sigma^{\mu\nu}\chi)} = \bar\chi\bar\sigma^{\mu\nu}\bar\psi \ .$$

ε-tensor: $\varepsilon_{\mu\nu\rho\lambda} \equiv \varepsilon_{abcd}{\rm e}_\mu{}^a{\rm e}_\nu{}^b{\rm e}_\rho{}^c{\rm e}_\lambda{}^d \ , \qquad \varepsilon_{0123} = +1 \ .$

Lie derivative: The Lie derivative $\mathcal{L}_\xi$ along ξ^μ is defined, for a scalar or a vector field, by:

$$\mathcal{L}_\xi\varphi = \xi^\mu\partial_\mu\varphi \tag{B.6}$$
$$\mathcal{L}_\xi\varphi_\nu = \xi^\mu\partial_\mu\varphi_\nu + \partial_\nu\xi^\mu\varphi_\mu \tag{B.7}$$
$$\mathcal{L}_\xi\varphi^\nu = \xi^\mu\partial_\mu\varphi^\nu - \partial_\mu\xi^\nu\varphi^\mu \ , \tag{B.8}$$

with the obvious generalization to the higher rank tensors. For a tensorial density $\Phi^{\cdots}$, the formula

$$\mathcal{L}_\xi\Phi^{\cdots} = e\mathcal{L}_\xi\left(e^{-1}\Phi^{\cdots}\right) + \partial_\mu(\xi^\mu e)e^{-1}\Phi^{\cdots} \ ,$$

defines its Lie derivative from the Lie derivative of the tensor $e^{-1}\Phi^{\cdots}$.

C. Vierbein formalism

As already said, we suppose that the space-time torsion is identically zero, which means that the Lorentz connection $w_{\mu ab}$ is expressed as a function of the vierbein ${\rm e}_\mu{}^a$:

$$w_{\mu ab} = \frac{1}{2}\left({\rm e}_{\nu a}\partial_\mu{\rm e}_b{}^\nu - {\rm e}_{\nu b}\partial_\mu{\rm e}_a{}^\nu - {\rm e}_a{}^\nu{\rm e}_b{}^\rho(\partial_\nu g_{\rho\mu} - \partial_\rho g_{\nu\mu})\right) \ . \tag{C.1}$$

The indices $\mu,\nu,\cdots = 0,\cdots,3$ and $a,b,\cdots = 0,\cdots,3$ are world and tangent space indices, respectively.

The inverse vierbein is given by ${\rm e}_a{}^\mu$, with:

$${\rm e}_a{}^\mu{\rm e}_\mu{}^b = \delta_a^b \ , \qquad\qquad {\rm e}_\mu{}^a{\rm e}_a{}^\nu = \delta_\mu^\nu \ . \tag{C.2}$$

The metric is then given by:

$$g_{\mu\nu} = {\rm e}_\mu{}^a{\rm e}_{\nu a} \ . \tag{C.3}$$

The metric $g_{\mu\nu}$ is used to raise and lower world indices, the flat metric η_{ab} is used to raise and lower the tangent space indices and the vierbein $\mathrm{e}_\mu{}^a$ is used to transform a tangent space indice into a world indice and vice versa:

$$
\begin{aligned}
g_{\mu\nu}\varphi^\nu &= \varphi_\mu \ , & \qquad g^{\mu\nu}\varphi_\nu &= \varphi^\mu \ , \\
\eta_{ab}\varphi^b &= \varphi_a \ , & \qquad \eta^{ab}\varphi_b &= \varphi^a \ , \\
\mathrm{e}_\mu{}^a\varphi_a &= \varphi_\mu \ , & \qquad \mathrm{e}_a{}^\mu\varphi_\mu &= \varphi_a \ ,
\end{aligned}
\tag{C.4}
$$

where $g^{\mu\nu}$ is the inverse metric.

The Christoffel symbols are defined by:

$$
\Gamma_\mu{}^\lambda{}_\nu = \frac{1}{2} g^{\lambda\rho}\left(\partial_\rho g_{\mu\nu} - \partial_\mu g_{\rho\nu} - \partial_\nu g_{\mu\rho}\right) \ . \tag{C.5}
$$

And finally, the Riemann tensor, the Ricci tensor and the scalar curvature are defined by:

$$
\begin{aligned}
R_{\mu\nu ab} &= \partial_\nu w_{\nu ab} - \partial_\mu w_{\mu ab} + w_{\nu a}{}^c w_{\mu cb} - w_{\mu a}{}^c w_{\nu cb} \ , \\
R_{\mu\nu} &= R_{\mu\lambda ab}\mathrm{e}^{b\lambda}\mathrm{e}_\nu{}^a \ , \\
R &= g^{\mu\nu}R_{\mu\nu} \ .
\end{aligned}
\tag{C.6}
$$

D. Covariant derivative

A generic field φ is completely characterized by

- its world indices: $\mu, \nu, \rho, \ldots$
- its Lorentz indices: $\begin{cases} a, b, c, \ldots & \text{tangent space vectorial indices} \\ \alpha, \beta, \gamma, \ldots & \text{undotted spinorial indices} \\ \dot\alpha, \dot\beta, \dot\gamma, \ldots & \text{dotted spinorial indices} \end{cases}$
- its $\mathcal{G}$-gauge indices: $\begin{cases} i, j, k, \ldots & \text{of the adjoint representation of } \mathcal{G} \\ A, B, C, \ldots & \text{of some rep. } R \text{ of } \mathcal{G} \end{cases}$
- its R-weight: R_φ
- its canonical dimension: d_φ .

(D.1)

The covariant derivative $\nabla_\mu\varphi$ of a tensor φ is then defined by

$$
\begin{aligned}
\nabla_\mu\varphi = \partial_\mu\varphi & \\
& +\Gamma_\mu{}^\lambda{}_\nu\varphi_\lambda && \text{for each covariant indice } \nu \text{ of } \varphi_\nu \\
& -\Gamma_\mu{}^\nu{}_\lambda\varphi^\lambda && \text{for each contravariant indice } \nu \text{ of } \varphi^\nu \\
& +w_{\mu a}{}^b\varphi_b && \text{for each tangent space Lorentz indice } a \text{ of } \varphi_a \\
& +\tfrac{i}{4}w_{\mu ab}\varphi^\beta\sigma^{ab}{}_\beta{}^\alpha && \text{for each undotted Lorentz indice } \alpha \text{ of } \varphi^\alpha \\
& -\tfrac{i}{4}w_{\mu ab}\bar\sigma^{ab\dot\beta}{}_{\dot\gamma}\varphi^{\dot\gamma} && \text{for each dotted Lorentz indice } \dot\beta \text{ of } \varphi^{\dot\beta} \\
& +f^{ijk}A_\mu^j\varphi^k && \text{for each gauge indice } i \text{ of } \varphi^i \\
& +T^i_{AB}A^i_\mu\varphi_B && \text{for each gauge indice } A \text{ of } \varphi_A \\
& -iR_\varphi B_\mu\varphi
\end{aligned}
\tag{D.2}
$$

E. BRS operator in the new formulation

The BRS operator s including all the symmetries of the new formulation of the model is given by

$$
\begin{aligned}
sA^i_\mu &= \mathcal{L}_\xi A^i_\mu + \nabla_\mu c^i + \varepsilon\sigma_\mu\bar\lambda^i + \lambda^i\sigma_\mu\bar\varepsilon \\
s\lambda^i &= \mathcal{L}_\xi\lambda^i - f^{ijk}c^j\lambda^k + \frac{i}{4}\lambda_{ab}\lambda^i\sigma^{ab} - i\eta\lambda^i - \frac{1}{2}\varepsilon\sigma^{\mu\nu}\widetilde{F}^i_{\mu\nu} - \frac{i}{2}G^2(\bar\phi_A T^i_{AB}\phi_B)\varepsilon + \frac{3}{2}\kappa\lambda^i \\
s\bar\lambda^i &= \mathcal{L}_\xi\bar\lambda^i - f^{ijk}c^j\bar\lambda^k - \frac{i}{4}\lambda_{ab}\bar\sigma^{ab}\bar\lambda^i + i\eta\bar\lambda^{i\dot\beta} + \frac{1}{2}\bar\sigma^{\mu\nu}\bar\varepsilon\widetilde{\bar F}^i_{\mu\nu} + \frac{i}{2}G^2(\bar\phi_A T^i_{AB}\phi_B)\bar\varepsilon + \frac{3}{2}\kappa\bar\lambda^i \\
s\phi_A &= \mathcal{L}_\xi\phi_A - T^i_{AB}c^i\phi_B - \frac{2}{3}i\eta\phi_A + 2\varepsilon\psi_A + \kappa\phi_A \\
s\bar\phi_A &= \mathcal{L}_\xi\bar\phi_A - T^i_{AB}c^i\bar\phi_B + \frac{2}{3}i\eta\bar\phi_A - 2\bar\varepsilon\bar\psi_A + \kappa\bar\phi_A \\
s\psi_A &= \mathcal{L}_\xi\psi_A - T^i_{AB}c^i\psi_B + \frac{i}{4}\lambda_{ab}\psi_A\sigma^{ab} \\
&\quad + \frac{1}{3}i\eta\psi_A + i\bar\varepsilon\bar\sigma^\mu\mathcal{D}_\mu\phi_A + 2\bar\Lambda_{(ABC)}\bar\phi_B\bar\phi_C\varepsilon + \frac{3}{2}\kappa\psi_A + 2\phi_A\chi \\
s\bar\psi_A &= \mathcal{L}_\xi\bar\psi_A - T^i_{AB}c^i\bar\psi_B - \frac{i}{4}\lambda_{ab}\bar\sigma^{ab}\bar\psi_A \\
&\quad - \frac{1}{3}i\eta\bar\psi_A + i\bar\sigma^\mu\varepsilon\mathcal{D}_\mu\bar\phi_A - 2\Lambda_{(ABC)}\phi_B\phi_C\bar\varepsilon + \frac{3}{2}\kappa\bar\psi_A - 2\bar\phi_A\bar\chi \\
sc^i &= \mathcal{L}_\xi c^i - \frac{1}{2}f^{ijk}c^jc^k - 2iE^\mu A^i_\mu \\
s\bar c^i &= \mathcal{L}_\xi\bar c^i + b^i + 2\kappa\bar c^i \\
sb^i &= \mathcal{L}_\xi b^i - 2iE^\mu\partial_\mu\bar c^i + 2\kappa b^i - 4(\bar\chi\bar\varepsilon + \chi\varepsilon)\bar c^i
\end{aligned}
\tag{E.1}
$$

$$
\begin{aligned}
s\mathrm{e}_\mu{}^a &= \mathcal{L}_\xi \mathrm{e}_\mu{}^a + \lambda^a{}_b \mathrm{e}_\mu{}^b + 2\varepsilon\sigma^a\bar{\Psi}_\mu + 2\Psi_\mu\sigma^a\bar{\varepsilon} - \kappa \mathrm{e}_\mu{}^a \\
s\Psi_\mu &= \mathcal{L}_\xi\Psi_\mu + \frac{i}{4}\lambda_{ab}\Psi_\mu\sigma^{ab} - i\eta\Psi_\mu - i\mathcal{D}_\mu\varepsilon - \frac{1}{2}\kappa\Psi_\mu + \bar{\chi}\bar{\sigma}_\mu \\
s\bar{\Psi}_\mu &= \mathcal{L}_\xi\bar{\Psi}_\mu - \frac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\Psi}_\mu + i\eta\bar{\Psi}_\mu - i\mathcal{D}_\mu\bar{\varepsilon} - \frac{1}{2}\kappa\bar{\Psi}_\mu + \chi\sigma_\mu \\
sC_{\mu\nu} &= \mathcal{L}_\xi C_{\mu\nu} + K_{\mu\nu} - 2\kappa C_{\mu\nu} \\
&\quad -2\varepsilon\sigma_\mu\bar{\Psi}_\nu - 2\Psi_\nu\sigma_\mu\bar{\varepsilon} + 2\varepsilon\sigma_\nu\bar{\Psi}_\mu + 2\Psi_\mu\sigma_\nu\bar{\varepsilon} \\
sB_\mu &= \mathcal{L}_\xi B_\mu + \partial_\mu\eta - \varepsilon\sigma_\mu\bar{\sigma}^{\nu\rho}\mathcal{D}_\nu\bar{\Psi}_\rho - \mathcal{D}_\nu\Psi_\rho\sigma^{\nu\rho}\sigma_\mu\bar{\varepsilon} \\
&\quad -3\left(\bar{\chi}\bar{\sigma}^\nu\sigma_\mu\bar{\Psi}_\nu\right) + 3\left(\chi\sigma^\nu\bar{\sigma}_\mu\Psi_\nu\right) - \frac{3}{8}\left(M_\mu - 2N_\mu\right)
\end{aligned} \tag{E.2}
$$

$$
\begin{aligned}
sI_\mu &= \mathcal{L}_\xi I_\mu + J_\mu \\
sJ_\mu &= \mathcal{L}_\xi J_\mu - 2iE^\nu\partial_\nu I_\mu - 2i\partial_\mu E^\nu I_\nu
\end{aligned} \tag{E.3}
$$

$$
\begin{aligned}
s\xi^\mu &= \xi^\lambda\partial_\lambda\xi^\mu + 2iE^\mu \\
s\lambda_{ab} &= \mathcal{L}_\xi\lambda_{ab} + (\lambda^2)_{ab} + 2iE^\lambda\widetilde{w}_{\lambda ab} - \frac{i}{2}\varepsilon_{ab\mu\nu}E^\mu V^\nu - 2i\left(\bar{\chi}\bar{\sigma}_{ab}\bar{\varepsilon}\right) - 2i\left(\chi\sigma_{ab}\varepsilon\right) \\
s\varepsilon &= \mathcal{L}_\xi\varepsilon + \frac{i}{4}\lambda_{ab}\varepsilon\sigma^{ab} - i\eta\varepsilon + 2E^\lambda\Psi_\lambda - \frac{1}{2}\kappa\varepsilon \\
s\bar{\varepsilon} &= \mathcal{L}_\xi\bar{\varepsilon} - \frac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\varepsilon} + i\eta\bar{\varepsilon} + 2E^\lambda\bar{\Psi}_\lambda - \frac{1}{2}\kappa\bar{\varepsilon} \\
s\eta &= \mathcal{L}_\xi\eta - 2iE^\lambda B_\lambda - 3i\left(\bar{\chi}\bar{\varepsilon} - \chi\varepsilon\right) \\
sK_{\mu\nu} &= \mathcal{L}_\xi K_{\mu\nu} - 2\kappa K_{\mu\nu} + 2i\partial_\mu\left(E_\nu + C_{\nu\rho}E^\rho\right) - 2i\partial_\nu\left(E_\mu + C_{\mu\rho}E^\rho\right) \\
&\quad +4i\left(\bar{\chi}\bar{\sigma}_{\mu\nu}\bar{\varepsilon} + \chi\sigma_{\mu\nu}\varepsilon\right) + 4\left(\bar{\chi}\bar{\varepsilon} + \chi\varepsilon\right)C_{\mu\nu} \\
s\chi &= \mathcal{L}_\xi\chi + \frac{i}{4}\lambda_{ab}\chi\sigma^{ab} + i\eta\chi + \frac{1}{2}\kappa\chi \\
&\quad -2\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\chi - 2\left(\bar{\varepsilon}\bar{\Psi}_\mu\right)\bar{\chi}\bar{\sigma}^\mu - \left(\frac{1}{8}\left(M_\mu - 2N_\mu\right) + \frac{i}{2}\partial_\mu\kappa\right)\bar{\varepsilon}\bar{\sigma}^\mu \\
s\bar{\chi} &= \mathcal{L}_\xi\bar{\chi} - \frac{i}{4}\lambda_{ab}\bar{\sigma}^{ab}\bar{\chi} - i\eta\bar{\chi} + \frac{1}{2}\kappa\bar{\chi} \\
&\quad -2\left(\varepsilon\sigma^\mu\bar{\Psi}_\mu\right)\bar{\chi} - 2\left(\varepsilon\Psi_\mu\right)\chi\sigma^\mu + \left(\frac{1}{8}\left(M_\mu - 2N_\mu\right) - \frac{i}{2}\partial_\mu\kappa\right)\varepsilon\sigma^\mu \\
s\kappa &= \mathcal{L}_\xi\kappa + 2\left(\bar{\chi}\bar{\varepsilon} + \chi\varepsilon\right)\ ,
\end{aligned} \tag{E.4}
$$

where we have used the notations (2.2, 2.4) and (4.5).

F. Classical breakings, algebra and Ward operators for the linear symmetries

We give in this appendix the classical breakings of the Ward identities defining the model, as well as the algebra formed by the ST operator and the Ward operators, which leads to the Ward operators for $\mathcal{G}$-rigid transformations, diffeomorphisms, Lorentz transformations, R- and C-transformations and, for the new formulation of the model, to the Ward operators associated with the shift identity, the supertrace identities and the Weyl identity. The distinction between old and new formulation is made whenever it is necessary.

We start by giving explicitly the classical breakings of the Ward identities (2.31) and (2.32) for the old formulation or (4.15) in the new formulation:

$$\begin{aligned}
\Delta^i_{\rm G} &= \int d^4x \Bigg(f^{ijk} \left\{ -A^{*j\mu} A^k_\mu + \lambda^{*j}\lambda^k + \bar\lambda^{*j}\bar\lambda^k + c^{*j}c^k \right\} \\
&\qquad\qquad + T^i_{AB} \left\{ \phi^*_A \phi_B + \bar\phi^*_A \bar\phi_B - \psi^*_A\psi_B - \bar\psi^*_A \bar\psi_B \right\} \Bigg) , \\
\Omega_{{\rm D}\mu} &= \sum_{\varphi\in I} (-1)^{{\rm gh}(\varphi^*)} \varphi^* \partial_\mu \varphi + \partial_\nu \left(A^{*i\nu} A^i_\mu \right) , \\
\Omega^{ab}_{\rm L} &= -\frac{i}{2}\lambda^{*i}\sigma^{ab}\lambda^i - \frac{i}{2}\bar\lambda^{*i}\sigma^{ab}\bar\lambda^i - \frac{i}{2}\psi^*_A\sigma^{ab}\psi_A - \frac{i}{2}\bar\psi^*_A\sigma^{ab}\bar\psi_A , \\
\Omega_{\rm R} &= \sum_{\varphi\in I} (-1)^{{\rm gh}(\varphi^*)} \varphi^* i R_\varphi \varphi .
\end{aligned} \tag{F.1}$$

For the supplementary ghost identities of the new formulation (4.16), the classical breakings are:

$$\begin{aligned}
\Omega_\chi &= -2\phi_A \psi^*_A , \\
\bar\Omega_{\bar\chi} &= -2\bar\phi_A \bar\psi^*_A , \\
\Omega_\kappa &= \sum_{\varphi\in I} (-1)^{{\rm gh}(\varphi^*)} \varphi^* \tilde d_\varphi \varphi .
\end{aligned} \tag{F.2}$$

We can then write down the nontrivial commutation rules for the algebra formed by the ST operator (2.24) and the operators defining the Ward identities (2.30-2.32) or (4.15-4.16), valid for any functional γ with zero GP:

$$\mathcal{B}_\gamma S(\gamma) = 0 , \tag{F.3}$$

$$\frac{\delta}{\delta b^i} S(\gamma) - \mathcal{B}_\gamma \left(\frac{\delta\gamma}{\delta b^i} - \Omega^i_b \right) = \bar F^i \gamma + b\Omega^i_b , \tag{F.4}$$

$$F^i S(\gamma) + \mathcal{B}_\gamma \left(F^i\gamma - \Delta^i_{\rm G} \right) = W^i_{\rm G} \gamma , \tag{F.5}$$

$$\frac{\delta}{\delta\xi^\mu} S(\gamma) + \mathcal{B}_\gamma \left(\frac{\delta\gamma}{\delta\xi^\mu} - \Omega_{{\rm D}\mu} \right) = \omega_{{\rm D}\mu}\gamma + \partial_\nu \left(\xi^\nu \frac{\delta\gamma}{\delta\xi^\mu} \right) , \tag{F.6}$$

$$\frac{\delta}{\delta\lambda_{ab}} S(\gamma) + \mathcal{B}_\gamma \left(\frac{\delta\gamma}{\delta\lambda_{ab}} - \Omega^{ab}_{\rm L} \right) = \omega^{ab}_{\rm L}\gamma + \partial_\mu \left(\xi^\mu \frac{\delta\gamma}{\delta\lambda_{ab}} \right) , \tag{F.7}$$

$$\frac{\delta}{\delta\eta} S(\gamma) + \mathcal{B}_\gamma \left(\frac{\delta\gamma}{\delta\eta} - \Omega_{\rm R} \right) = \omega_{\rm R}\gamma + \partial_\mu \left(\xi^\mu \frac{\delta\gamma}{\delta\eta} \right) . \tag{F.8}$$

Moreover, for the old formulation:

$$\frac{\delta}{\delta w_\mu} S(\gamma) + \mathcal{B}_\gamma \left(\frac{\delta\gamma}{\delta w_\mu} \right) = \omega^\mu_{\rm C}\gamma + \partial_\nu \left(\xi^\nu \frac{\delta\gamma}{\delta w_\mu} \right) - \partial_\nu \xi^\mu \frac{\delta\gamma}{\delta w_\nu} - 2iE^\mu \frac{\delta\gamma}{\delta w} , \tag{F.9}$$

$$\frac{\delta}{\delta w}S(\gamma)-\mathcal{B}_\gamma\left(\frac{\delta\gamma}{\delta w}\right)=-\partial_\mu\frac{\delta\gamma}{\delta w_\mu}-\partial_\mu\left(\xi^\mu\frac{\delta\gamma}{\delta w}\right)\ , \tag{F.10}$$

whereas, for the new formulation:

$$\begin{aligned}\frac{\delta}{\delta K_{\mu\nu}}S(\gamma)+\mathcal{B}_\gamma\left(\frac{\delta\gamma}{\delta K_{\mu\nu}}\right)&=\omega_K^{\mu\nu}\gamma+\partial_\rho\left(\xi^\rho\frac{\delta\gamma}{\delta K_{\mu\nu}}\right)\\&-\partial_\rho\xi^\mu\frac{\delta\gamma}{\delta K_{\rho\nu}}+\partial_\rho\xi^\nu\frac{\delta\gamma}{\delta K_{\rho\mu}}+2\kappa\frac{\delta\gamma}{\delta K_{\mu\nu}}\ ,\end{aligned} \tag{F.11}$$

$$\begin{aligned}\frac{\delta}{\delta\chi}S(\gamma)-\mathcal{B}_\gamma\left(\frac{\delta\gamma}{\delta\chi}-\Omega_\chi\right)&=\omega_\chi\gamma-\partial_\mu\left(\xi^\mu\frac{\delta\gamma}{\delta\chi}\right)\\&+\left(\frac{i}{4}\lambda_{ab}\sigma^{ab}+i\eta+\frac{1}{2}\kappa-2\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\right)\frac{\delta\gamma}{\delta\chi}\ ,\end{aligned} \tag{F.12}$$

$$\mathcal{H}_\kappa S(\gamma)+\mathcal{B}_\gamma\left(\mathcal{H}_\kappa\gamma-\Omega_\kappa\right)=\omega_\kappa\gamma+\partial_\mu\left(\xi^\mu\ \mathcal{H}_\kappa\gamma\right)\ . \tag{F.13}$$

This algebra implies the following Ward operators:

The antighost operator

For the old formulation:

$$\bar{F}^i=\frac{\delta}{\delta\bar{c}^i}+\partial_\mu\left(\sqrt{g}g^{\mu\nu}\frac{\delta}{\delta A^{*i\nu}}\right)+2\kappa\frac{\delta}{\delta b^i}-\partial_\mu\left(\xi^\mu\frac{\delta}{\delta b^i}\right)\ , \tag{F.14}$$

and for the new formulation:

$$\bar{F}^i=\frac{\delta}{\delta\bar{c}^i}+\left(\partial_\mu-2I_\mu\right)\left(\sqrt{g}g^{\mu\nu}\frac{\delta}{\delta A^{*i\nu}}\right)+2\kappa\frac{\delta}{\delta b^i}-\partial_\mu\left(\xi^\mu\frac{\delta}{\delta b^i}\right)\ . \tag{F.15}$$

Ward operator for the rigid $\mathcal{G}$-transformations

$$\begin{aligned}W^i_{\rm G}=\int d^4x\Bigg(&-f^{jik}\left\{A^k_\mu\frac{\delta}{\delta A^j_\mu}+A^{*k\mu}\frac{\delta}{\delta A^{*j\mu}}+\lambda^k\frac{\delta}{\delta\lambda^j}+\lambda^{*k}\frac{\delta}{\delta\lambda^{*j}}\right.\\&\left.-\bar{\lambda}^k\frac{\delta}{\delta\bar{\lambda}^j}-\bar{\lambda}^{*k}\frac{\delta}{\delta\bar{\lambda}^{*j}}+c^k\frac{\delta}{\delta c^j}+c^{*k}\frac{\delta}{\delta c^{*j}}+\bar{c}^k\frac{\delta}{\delta\bar{c}^j}+b^k\frac{\delta}{\delta b^j}\right\}\\&-T^i_{AB}\left\{\phi_B\frac{\delta}{\delta\phi_A}+\phi^*{}_B\frac{\delta}{\delta\phi^*{}_A}+\bar{\phi}_B\frac{\delta}{\delta\bar{\phi}_A}+\bar{\phi}^*{}_B\frac{\delta}{\delta\bar{\phi}^*{}_A}+\psi_B\frac{\delta}{\delta\psi_A}+\psi^*_B\frac{\delta}{\delta\psi^*_A}\right.\\&\left.-\bar{\psi}_B\frac{\delta}{\delta\bar{\psi}_A}-\bar{\psi}^*{}_B\frac{\delta}{\delta\bar{\psi}^*{}_A}\right\}\Bigg)\ .\end{aligned} \tag{F.16}$$

Ward operator for the diffeomorphisms

$$\omega_{\mathrm{D}\mu} = \omega_{\mathrm{T}\mu} + \partial_\nu \omega_\mu{}^\nu \ , \tag{F.17}$$

where

$$\omega_{\mathrm{T}\mu} = \left(\sum_{\text{all fields}} \partial_\mu \varphi \frac{\delta}{\delta \varphi} \right) \ , \tag{F.18}$$

and, for the old formulation,

$$\begin{aligned} \omega_\mu{}^\nu = & \left(\delta^\nu_\mu \sum_{\varphi^* \in I^*} \varphi^* \frac{\delta}{\delta \varphi^*} - A^i_\mu \frac{\delta}{\delta A^i_\nu} + A^{*i\nu} \frac{\delta}{\delta A^{*i\mu}} - \mathrm{e}_\mu{}^a \frac{\delta}{\delta \mathrm{e}_\nu{}^a} \right. \\ & \left. - \Psi_\mu \frac{\delta}{\delta \Psi_\nu} + \bar\Psi_\mu \frac{\delta}{\delta \bar\Psi_\nu} - C_{\mu\rho} \frac{\delta}{\delta C_{\nu\rho}} - B_\mu \frac{\delta}{\delta B_\nu} - w_\mu \frac{\delta}{\delta w_\nu} \right) \ , \end{aligned} \tag{F.19}$$

whereas for the new formulation:

$$\begin{aligned} \omega_\mu{}^\nu = & \left(\delta^\nu_\mu \sum_{\varphi^* \in I^*} \varphi^* \frac{\delta}{\delta \varphi^*} - A^i_\mu \frac{\delta}{\delta A^i_\nu} + A^{*i\nu} \frac{\delta}{\delta A^{*i\mu}} - \mathrm{e}_\mu{}^a \frac{\delta}{\delta \mathrm{e}_\nu{}^a} \right. \\ & - \Psi_\mu \frac{\delta}{\delta \Psi_\nu} + \bar\Psi_\mu \frac{\delta}{\delta \bar\Psi_\nu} - C_{\mu\rho} \frac{\delta}{\delta C_{\nu\rho}} \\ & \left. - B_\mu \frac{\delta}{\delta B_\nu} - K_{\mu\rho} \frac{\delta}{\delta K_{\nu\rho}} - I_\mu \frac{\delta}{\delta I_\nu} - J_\mu \frac{\delta}{\delta J_\nu} \right) \ . \end{aligned} \tag{F.20}$$

Ward operator for the Lorentz transformations

For the old formulation:

$$\begin{aligned} \omega^{ab}_{\mathrm{L}} = & \left(\frac{i}{2} \lambda^i \sigma^{ab} \frac{\delta}{\delta \lambda^i} - \frac{i}{2} \bar\lambda^i \sigma^{ab} \frac{\delta}{\delta \bar\lambda^i} + \frac{i}{2} \psi_A \sigma^{ab} \frac{\delta}{\delta \psi_A} - \frac{i}{2} \bar\psi_A \sigma^{ab} \frac{\delta}{\delta \bar\psi_A} \right. \\ & + \left(\eta^{ac} \mathrm{e}_\mu{}^b - \eta^{bc} \mathrm{e}_\mu{}^a \right) \frac{\delta}{\delta \mathrm{e}_\mu{}^c} + \frac{i}{2} \Psi_\mu \sigma^{ab} \frac{\delta}{\delta \Psi_\mu} - \frac{i}{2} \bar\Psi_\mu \sigma^{ab} \frac{\delta}{\delta \bar\Psi_\mu} \\ & \left. + \frac{i}{2} \varepsilon \sigma^{ab} \frac{\delta}{\delta \varepsilon} - \frac{i}{2} \bar\varepsilon \sigma^{ab} \frac{\delta}{\delta \bar\varepsilon} + 2 \left(\delta^a_c \lambda^b{}_d - \delta^b_c \lambda^a{}_d \right) \frac{\delta}{\delta \lambda_{cd}} \right) \ , \end{aligned} \tag{F.21}$$

and for the new formulation:

$$\begin{aligned} \omega^{ab}_{\mathrm{L}} = & \left(\frac{i}{2} \lambda^i \sigma^{ab} \frac{\delta}{\delta \lambda^i} - \frac{i}{2} \bar\lambda^i \sigma^{ab} \frac{\delta}{\delta \bar\lambda^i} + \frac{i}{2} \psi_A \sigma^{ab} \frac{\delta}{\delta \psi_A} - \frac{i}{2} \bar\psi_A \sigma^{ab} \frac{\delta}{\delta \bar\psi_A} \right. \\ & + \left(\eta^{ac} \mathrm{e}_\mu{}^b - \eta^{bc} \mathrm{e}_\mu{}^a \right) \frac{\delta}{\delta \mathrm{e}_\mu{}^c} + \frac{i}{2} \Psi_\mu \sigma^{ab} \frac{\delta}{\delta \Psi_\mu} - \frac{i}{2} \bar\Psi_\mu \sigma^{ab} \frac{\delta}{\delta \bar\Psi_\mu} \\ & + \frac{i}{2} \varepsilon \sigma^{ab} \frac{\delta}{\delta \varepsilon} - \frac{i}{2} \bar\varepsilon \sigma^{ab} \frac{\delta}{\delta \bar\varepsilon} + 2 \left(\delta^a_c \lambda^b{}_d - \delta^b_c \lambda^a{}_d \right) \frac{\delta}{\delta \lambda_{cd}} \\ & \left. + \frac{i}{2} \chi \sigma^{ab} \frac{\delta}{\delta \chi} - \frac{i}{2} \bar\chi \sigma^{ab} \frac{\delta}{\delta \bar\chi} \right) \ . \end{aligned} \tag{F.22}$$

Ward operator for the R-transformations

$$\omega_{\rm R} = \left(-\partial_\mu \frac{\delta}{\delta B_\mu} + \sum_{\text{all fields}} iR_\varphi \varphi \frac{\delta}{\delta\varphi} \right) . \tag{F.23}$$

Ward operators for the C-transformations

(old formulation only)

$$\omega^\mu_{\rm C} = \left(\partial_\nu \frac{\delta}{\delta C_{\mu\nu}} \right) . \tag{F.24}$$

Ward operator for the shift transformations

(new formulation only)

$$\omega^{\mu\nu}_K = \left(\frac{\delta}{\delta C_{\mu\nu}} + \frac{1}{4}\partial_\lambda \left\{ \varepsilon_\rho{}^{\lambda\mu\nu} \left(3\frac{\delta}{\delta B_\rho} + \bar\varepsilon\bar\sigma^\rho \frac{\delta}{\delta\chi} + \varepsilon\sigma^\rho \frac{\delta}{\delta\bar\chi} \right) \right\} \right) . \tag{F.25}$$

Ward operator for the supertrace transformations

(new formulation only)

$$\begin{aligned} \omega_\chi = \Bigg(& 2\phi_A \frac{\delta}{\delta\psi_A} - 2\psi^*_A \frac{\delta}{\delta\phi^*_A} - \sigma_\mu \frac{\delta}{\delta\bar\Psi_\mu} + 3\sigma^\nu\bar\sigma_\mu\Psi_\nu \frac{\delta}{\delta B_\mu} - i\sigma_{ab}\varepsilon \frac{\delta}{\delta\lambda_{ab}} \\ & -4\varepsilon\bar c^i \frac{\delta}{\delta b^i} + 3i\varepsilon\frac{\delta}{\delta\eta} + 2\varepsilon\frac{\delta}{\delta\kappa} + 2\left(\varepsilon\Psi_\mu\right)\sigma^\mu \frac{\delta}{\delta\bar\chi} + \left(4i\sigma_{\mu\nu}\varepsilon + 4\varepsilon C_{\mu\nu}\right)\frac{\delta}{\delta K_{\mu\nu}} \Bigg) \end{aligned} \tag{F.26}$$

Ward operator for the (modified) Weyl transformations

(new formulation only)

$$\begin{aligned} \omega_\kappa = \Bigg(& \sum_{\text{all fields}} \tilde d_\varphi \varphi \frac{\delta}{\delta\varphi} - \frac{1}{4}\partial_\lambda \left\{ \varepsilon_\rho{}^{\lambda\mu\nu} C_{\mu\nu} \left(3\frac{\delta}{\delta B_\rho} + \bar\varepsilon\bar\sigma^\rho \frac{\delta}{\delta\chi} + \varepsilon\sigma^\rho \frac{\delta}{\delta\bar\chi} \right) \right\} \\ & + \partial_\mu \left\{ \frac{i}{2}\bar\varepsilon\bar\sigma^\mu \frac{\delta}{\delta\chi} - \frac{i}{2}\varepsilon\sigma^\mu \frac{\delta}{\delta\bar\chi} + \frac{\delta}{\delta I_\mu} \right\} \Bigg) . \end{aligned} \tag{F.27}$$

Linear Symmetries

Finally, setting $\gamma = \Sigma$ in (F.4) to (F.13) leads to

$$\begin{aligned} & \bar F^i \Sigma = -b\Omega^i_b = \Omega^i_{\bar c} \, , \quad W^i_{\rm G}\Sigma = 0 \, , \\ & \omega_{{\rm D}\mu}\Sigma = -\partial_\nu\left(\xi^\nu \Omega_{{\rm D}\mu}\right) \, , \quad \omega^{ab}_{\rm L}\Sigma = -\partial_\mu\left(\xi^\mu\Omega^{ab}_{\rm L}\right) \, , \quad \omega_{\rm R}\Sigma = -\partial_\mu\left(\xi^\mu\Omega_{\rm R}\right) \, , \end{aligned} \tag{F.28}$$

and, for the old formulation:

$$\omega^\mu_{\rm C}\Sigma = 0 \, , \tag{F.29}$$

whereas for the new formulation:

$$
\begin{aligned}
&\omega_K^{\mu\nu}\Sigma = 0 \ , \\
&\omega_\chi\Sigma = \partial_\mu\left(\xi^\mu\Omega_\chi\right) - \left(\frac{i}{4}\lambda_{ab}\sigma^{ab} + i\eta + \frac{1}{2}\kappa - 2\left(\Psi_\mu\sigma^\mu\bar{\varepsilon}\right)\right)\Omega_\chi \ , \\
&\omega_\kappa\Sigma = -\partial_\mu\left(\xi^\mu\Omega_\kappa\right) \ .
\end{aligned}
\tag{F.30}
$$

These are the (linearly) broken classical symmetries of the model encoded in the ST identity.

Local Supersymmetry

There is no such simple Ward identity for local supersymmetry. This is due to the nonlinearity of the supersymmetry transformations and therefore to the absence of a ghost identity for the supersymmetry ghost ε. We have indeed seen that the Ward identities corresponding to the linear symmetries, a priori contained in the Slavnov-Taylor identity, are extracted therefrom through the action of the ghost identity operators, as explained at the beginning of this Appendix. It is however possible to write down a Ward identity for the local supersymmetry in the F.L. – hence for the conservation of the spinor current ${Q^\mu}_\alpha$ – which one may define [12] as the derivative of the Slavnov-Taylor identity with respect to $\varepsilon(x)$, setting then $\varepsilon(x)$ as well as the remaining external ghosts to zero:

$$
\begin{aligned}
\omega_{\mathrm{S}\alpha}\Sigma &= \left.\frac{\delta}{\delta\varepsilon^\alpha}\mathcal{S}(\Sigma)\right|_{\text{ext. ghosts}=0} \\
&= \left.\left(\Sigma, \frac{\delta\Sigma}{\delta\varepsilon^\alpha}\right)\right|_{\text{ext. ghosts}=0} + i\partial_\mu\frac{\delta\left.\Sigma\right|_{\text{ext. ghosts}=0}}{\delta\Psi_\mu^\alpha} \\
&\qquad + \text{terms vanishing in the flat limit} = 0 \ .
\end{aligned}
\tag{F.31}
$$

Rigid Ward Operators

By integrating these local Ward identities, we find the (non-broken) global Ward identities

$$
\begin{aligned}
&W_{\mathrm{D}\mu}\Sigma = 0 \ , \qquad W_{\mathrm{L}}^{ab}\Sigma = 0 \ , \qquad W_{\mathrm{R}}\Sigma = 0 \ , \\
&W_K^{\mu\nu}\Sigma = 0 \ , \qquad W_\kappa\Sigma = 0 \ ,
\end{aligned}
\tag{F.32}
$$

corresponding to invariance under the translations, the (global) Lorentz transformations, the (global) R-transformations, the (global) shift transformations and the (global) Weyl transformations, respectively.

G. Computation of Σ_c

In this Appendix, we solve the cohomological problem set by (5.2),

$$
\mathcal{B}_\Sigma\Sigma_c = 0 \ , \tag{G.1}
$$

in the space of local functionals in the fields, with the set of supplementary conditions (5.3)

$$\frac{\delta\Sigma_c}{\delta b^i}=0\ ,\qquad F^i\Sigma_c=0\ , \tag{G.2}$$

$$\frac{\delta\Sigma_c}{\delta\xi^\mu}=\frac{\delta\Sigma_c}{\delta\lambda_{ab}}=\frac{\delta\Sigma_c}{\delta\eta}=0\ , \tag{G.3}$$

$$\frac{\delta\Sigma_c}{\delta K_{\mu\nu}}=\frac{\delta\Sigma_c}{\delta\chi}=\frac{\delta\Sigma_c}{\delta\bar\chi}=0\ , \tag{G.4}$$

$$h_\kappa\Sigma_c=0\ . \tag{G.5}$$

The constraints (G.2-G.5) are easily solved and lead to a Σ_c which is independent of $b^i,\xi^\mu,\lambda_{ab},\eta,K_{\mu\nu},\chi,\bar\chi$ and which depends on c^i only through its derivative $\partial_\mu c^i$ and on κ only through the combination

$$\hat J_\mu=J_\mu+\partial_\mu\kappa\ . \tag{G.6}$$

We can obtain further constraints by using (G.2-G.5) and the algebra given in F. For example,

$$0=\frac{\delta}{\delta b^i}\mathcal{B}_\Sigma\Sigma_c-\mathcal{B}_\Sigma\frac{\delta}{\delta b^i}\Sigma_c=\bar F^i\Sigma_c\ . \tag{G.7}$$

In the same way, we obtain:

$$W^i_{\mathrm{G}}\Sigma_c=0\ , \tag{G.8}$$

$$\omega_{\mathrm{D}\mu}\Sigma_c=\omega^{ab}_{\mathrm{L}}\Sigma_c=\omega_{\mathrm{R}}\Sigma_c=0\ , \tag{G.9}$$

$$\omega^{\mu\nu}_K\Sigma_c=0\ , \tag{G.10}$$

$$\omega_\kappa\Sigma_c=0\ . \tag{G.11}$$

$$\omega_\chi\Sigma_c=\omega_{\bar\chi}\Sigma_c=0\ . \tag{G.12}$$

(G.7) and (G.10) are easily solved and lead to a Σ_c that depends on $\bar c^i$ and on $C_{\mu\nu}$ only through the combinations

$$\begin{aligned}\hat A^{*i\mu}&=A^{*i\mu}+\sqrt{g}g^{\mu\nu}(\partial_\nu+2I_\nu)\bar c^i\\ \hat B_\mu&=B_\mu+\tfrac{3}{8}\varepsilon_\mu{}^{\nu\rho\lambda}\partial_\nu C_{\rho\lambda}\ .\end{aligned} \tag{G.13}$$

In the new variables $\hat J_\mu$, $\hat A^{*i\mu}$ and $\hat B_\mu$, (G.5), (G.7) and (G.10) look much simpler:

$$\frac{\delta\Sigma_c}{\delta\kappa}=0\ , \tag{G.14}$$

$$\frac{\delta\Sigma_c}{\delta\bar c^i}=0\ , \tag{G.15}$$

$$\frac{\delta\Sigma_c}{\delta C_{\mu\nu}}=0\ , \tag{G.16}$$

and therefore we assume from now on to work with these new variables.

The general solution of (G.1) can be written formally as

$$\Sigma_c = \Sigma_{\rm ph} + \mathcal{B}_\Sigma \hat{\Sigma}_c \ . \tag{G.17}$$

We will proceed in two steps: the first to explicitly construct $\hat{\Sigma}_c$ by enumerating the candidates and applying the constraints and the second to find the cohomology element $\Sigma_{\rm ph}$ by filtration methods [6].

G.1. Construction of $\hat{\Sigma}_c$

We look for the most general functional $\hat{\Sigma}_c$ with dimension 4, ghost number -1 and R-weight 0 such that $\mathcal{B}_\Sigma \hat{\Sigma}_c$ satisfies (G.2)-(G.5) and (G.8)-(G.12).

$\hat{\Sigma}_c$ is independent of $\bar{c}^i$ and b^i: Let us introduce the counting operator

$$N = \int d^4x \left\{ b^i \frac{\delta}{\delta b^i} + \bar{c}^i \frac{\delta}{\delta \bar{c}^i} \right\} \ . \tag{G.18}$$

On one hand we have

$$N \left(\mathcal{B}_\Sigma \hat{\Sigma}_c \right) = 0 \ , \tag{G.19}$$

which follows form (G.2) and (G.15). On the other hand,

$$[N, \mathcal{B}_\Sigma] = 0 \ . \tag{G.20}$$

So, expanding $\hat{\Sigma}_c$ according to N, namely writing

$$\hat{\Sigma}_c = \sum_{n\geq 0} \hat{\Sigma}_c^{(n)} \qquad \text{with} \qquad N \hat{\Sigma}_c^{(n)} = n \hat{\Sigma}_c^{(n)} \ , \tag{G.21}$$

we have

$$0 = [N, \mathcal{B}_\Sigma]\, \hat{\Sigma}_c = -\mathcal{B}_\Sigma \left(N \hat{\Sigma}_c \right) = -\sum_{n\geq 0} n \ \mathcal{B}_\Sigma \hat{\Sigma}_c^{(n)} \ , \tag{G.22}$$

which means that only $\hat{\Sigma}_c^{(0)}$, *i.e.* the $b^i, \bar{c}^i$ independent part of $\hat{\Sigma}_c$, contributes to $\mathcal{B}_\Sigma \hat{\Sigma}_c$. Thus, without loss of generality, we can choose $\hat{\Sigma}_c$ independent of b^i and $\bar{c}^i$.

$\hat{\Sigma}_c$ is independent of $C_{\mu\nu}$: As $\mathcal{B}_\Sigma \hat{\Sigma}_c$ is constrained to be independent of $K_{\mu\nu}$ (see (G.4)), we have

$$\begin{aligned} 0 &= \frac{\delta}{\delta K_{\mu\nu}} \left(\mathcal{B}_\Sigma \hat{\Sigma}_c \right) = \left\{ \frac{\delta}{\delta K_{\mu\nu}}, \mathcal{B}_\Sigma \right\} \hat{\Sigma}_c - \mathcal{B}_\Sigma \frac{\delta \hat{\Sigma}_c}{\delta K_{\mu\nu}} \\ &= \omega_K^{\mu\nu} \hat{\Sigma}_c + \partial_\rho \left(\xi^\rho \frac{\delta \hat{\Sigma}_c}{\delta K_{\mu\nu}} \right) - \partial_\rho \xi^\mu \frac{\delta \hat{\Sigma}_c}{\delta K_{\rho\nu}} + \partial_\rho \xi^\nu \frac{\delta \hat{\Sigma}_c}{\delta K_{\rho\mu}} + 2\kappa \frac{\delta \hat{\Sigma}_c}{\delta K_{\mu\nu}} - \mathcal{B}_\Sigma \frac{\delta \hat{\Sigma}_c}{\delta K_{\mu\nu}} \ . \end{aligned} \tag{G.23}$$

It can be easily seen by writing down the most general candidate that $\hat{\Sigma}_c$ is independent of $K_{\mu\nu}, \chi$ and $\bar{\chi}$, and therefore, (G.23) leads to

$$\frac{\delta \hat{\Sigma}_c}{\delta C_{\mu\nu}} = 0 \ . \tag{G.24}$$

$\hat{\Sigma}_c$ is independent of c^i and satisfies $W^i_{\rm G}\hat{\Sigma}_c = 0$: We can write

$$\hat{\Sigma}_c = \int d^4x \, \left(Q^{ij}({\rm e}_\mu{}^a) c^{*i} c^j + \text{ independent of } c^i \right) \, , \tag{G.25}$$

where the scalar $Q^{ij}({\rm e}_\mu{}^a)$ is a local functional in the vierbein. From (G.2), we get

$$\int d^4x \, \frac{\delta}{\delta c^i} \left(\mathcal{B}_\Sigma \hat{\Sigma}_c \right) = 0 \, , \tag{G.26}$$

which leads, by an explicit calculation of the term linear in c^i, to $Q^{ij}({\rm e}_\mu{}^a) = 0$. Thus, $\hat{\Sigma}_c$ is independent of c^i. Then, we write

$$0 = \int d^4x \, \frac{\delta}{\delta c^i} \left(\mathcal{B}_\Sigma \hat{\Sigma}_c \right) = \left\{ \int d^4x \, \frac{\delta}{\delta c^i}, \mathcal{B}_\Sigma \right\} \hat{\Sigma}_c - \mathcal{B}_\Sigma \int d^4x \, \frac{\delta \hat{\Sigma}_c}{\delta c^i} = W^i_{\rm G} \hat{\Sigma}_c \, , \tag{G.27}$$

which shows that $\hat{\Sigma}_c$ is invariant under the $\mathcal{G}$-rigid transformations.

$\hat{\Sigma}_c$ is independent of ξ^μ: We can write

$$\hat{\Sigma}_c = \int d^4x \, \left(P_\mu \xi^\mu + \text{ independent of } \xi^\mu \right) \, , \tag{G.28}$$

where P_μ is a functional of dimension 5, ghost number -2 and R-weight 0. From the condition $\dfrac{\delta}{\delta \xi^\mu} \mathcal{B}_\Sigma \hat{\Sigma}_c = 0$, we get $P_\mu = 0$ and thus $\hat{\Sigma}_c$ is independent of ξ^μ.

Remaining candidates: We can now list the remaining functionals potentially constituting $\hat{\Sigma}_c$

$$\begin{aligned} \hat{\Sigma}_c = \int d^4x \, \Bigg(& Z_A \hat{A}^{*i\mu} A^i_\mu + Z_\lambda \lambda^{*i} \lambda^i + \bar{Z}_{\bar{\lambda}} \bar{\lambda}^{*i} \bar{\lambda}^i + Z_{\phi AB} \phi^*_A \phi_B + \bar{Z}_{\bar{\phi} AB} \bar{\phi}^*_A \bar{\phi}_B \\ & + Z_{\psi AB} \psi^*_A \psi_B + \bar{Z}_{\bar{\psi} AB} \bar{\psi}^*_A \bar{\psi}_B + Y_{AB} \left(\bar{\Psi}_\mu \bar{\sigma}^\mu \psi^*_A \right) \phi_B - \bar{Y}_{AB} \left(\Psi_\mu \sigma^\mu \bar{\psi}^*_A \right) \bar{\phi}_B \\ & + V g^{\mu\rho} \left(\Psi_\mu + \frac{i}{3} \Psi_\nu \sigma^\nu{}_\mu \right) \lambda^{*i} A^i_\rho + W g^{\mu\rho} \left(\Psi_\mu \lambda^{*i} \right) A^i_\rho \\ & + \bar{V} g^{\mu\rho} \left(\bar{\Psi}_\mu + \frac{i}{3} \bar{\Psi}_\nu \bar{\sigma}^\nu{}_\mu \right) \bar{\lambda}^{*i} A^i_\rho + \bar{W} g^{\mu\rho} \left(\bar{\Psi}_\mu \bar{\lambda}^{*i} \right) A^i_\rho \Bigg) \, , \end{aligned} \tag{G.29}$$

where Z_A, Z_λ, V and W are arbitrary constants and $Z_{\phi AB}$, $Z_{\psi AB}$ and Y_{AB} are $\mathcal{G}$-invariant tensors.

The last step is to apply $\mathcal{B}_\Sigma$ on this $\hat{\Sigma}_c$ and to impose the remaining constraints, namely (G.4), which lead to

$$\begin{aligned} & W = \bar{W} = 0 \\ & Z_{\phi AB} + Z_{\psi AB} - 2Y_{AB} = 0 \\ & \bar{Z}_{\phi AB} + \bar{Z}_{\psi AB} - 2\bar{Y}_{AB} = 0 \, . \end{aligned} \tag{G.30}$$

Thus, finally, $\hat{\Sigma}_c$ is given by

$$\hat{\Sigma}_c = \int d^4x \left(Z_A A^i_\mu \hat{A}^{*i\mu} + Z_\lambda \lambda^{*i}\lambda^i + \bar{Z}_{\bar\lambda}\bar\lambda^{*i}\bar\lambda^i \right.$$
$$+ Z_{\phi AB}\left(\phi^*_A + \frac{1}{2}\bar\Psi_\mu\bar\sigma^\mu\psi^*_A\right)\phi_B + \bar{Z}_{\phi AB}\left(\bar\phi^*_A - \frac{1}{2}\Psi_\mu\sigma^\mu\bar\psi^*_A\right)\bar\phi_B$$
$$+ Z_{\psi AB}\psi^*_A\left(\psi_B + \frac{1}{2}\sigma^\mu\bar\Psi_\mu\phi_B\right) + \bar{Z}_{\psi AB}\bar\psi^*_A\left(\bar\psi_B - \frac{1}{2}\bar\sigma^\mu\Psi_\mu\bar\phi_B\right)$$
$$\left. + V g^{\mu\rho}\left(\Psi_\mu + \frac{i}{3}\Psi_\nu\sigma^\nu{}_\mu\right)\lambda^{*i}A^i_\rho + \bar{V} g^{\mu\rho}\left(\bar\Psi_\mu + \frac{i}{3}\bar\Psi_\nu\bar\sigma^\nu{}_\mu\right)\bar\lambda^{*i}A^i_\rho \right) \tag{G.31}$$

The interpretation of the different terms is given in the text, after (5.7).

G.2. Computation of the cohomology element $\Sigma_{\rm ph}$

Let us denote by $\mathcal{F}$ the space of local functionals in the fields with dimension 4, ghost number 0 and R-weight 0, constrained by (G.2-G.5) and (G.8-G.11)[10]. We are thus looking for the cohomology of $\mathcal{B}_\Sigma$ in $\mathcal{F}$, denoted by $\mathcal{H}(\mathcal{F},\mathcal{B}_\Sigma)$. To do this, let us introduce the filtration operator

$$N = \int d^4x \left\{ \varepsilon\frac{\delta}{\delta\varepsilon} - \bar\varepsilon\frac{\delta}{\delta\bar\varepsilon} + \Psi_\mu\frac{\delta}{\delta\Psi_\mu} - \bar\Psi_\mu\frac{\delta}{\delta\bar\Psi_\mu} + \chi\frac{\delta}{\delta\chi} - \bar\chi\frac{\delta}{\delta\bar\chi} \right\} , \tag{G.32}$$

which leads to the following decomposition for $\mathcal{B}_\Sigma$ and $\mathcal{F}$:

$$\begin{aligned} \mathcal{B}_\Sigma &= \sum_{k=0}^{4}\mathcal{B}^{(k)}_\Sigma , \qquad \text{with} \quad [N,\mathcal{B}^{(k)}_\Sigma] = k\mathcal{B}^{(k)}_\Sigma , \\ \mathcal{F} &= \sum_{k=0}^{8}\mathcal{F}^{(k)} , \qquad \text{with} \quad N\Xi^{(k)} = k\Xi^{(k)} \ \text{ if } \ \Xi^{(k)} \in \mathcal{F}^{(k)} . \end{aligned} \tag{G.33}$$

Writing down explicitly a basis of $\mathcal{F}$ shows that there is no element of order greater than 8.

The procedure we apply in order to find $\mathcal{H}(\mathcal{F},\mathcal{B}_\Sigma)$ was developed in [33] and applied, in particular, in [12, 10, 11, 13, 14, 25, 34]. It consists in the following steps:

Step 1: It has been shown in [33] that for each element of $\mathcal{H}(\mathcal{F},\mathcal{B}_\Sigma)$, we can pick up a particular representative which has the property that its lowest order according to the filtration N is in the cohomology of $\mathcal{B}^{(0)}_\Sigma$. Thus, the first step consists in constructing the cohomology of $\mathcal{B}^{(0)}_\Sigma$ in the different spaces $\mathcal{F}^{(k)}$, denoted by $\mathcal{H}(\mathcal{F}^{(k)},\mathcal{B}^{(0)}_\Sigma)$, for $k = 1,\ldots,8$.

[10](G.12) is not included in the set of constraints defining $\mathcal{F}$ since the filtration operator we are just going to introduce (G.32) does not commute with ω_χ. Nevertheless, as ω_χ is contained in the ST linearized operator $\mathcal{B}_\Sigma$, the cohomology elements $\Sigma_{\rm ph}$ we are going to construct will be invariant under ω_χ thanks to (G.1). This accounts for the fact that the constraints (G.8-G.12) are contained in the $\mathcal{B}_\Sigma$-invariance constraint (G.1) and thus do not need to be imposed separately.

Step 2: We then proceed to the construction of the "extension" $\mathcal{H}_{\rm ext}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)})$ of $\mathcal{H}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)})$, defined by

$$\begin{aligned}\mathcal{H}_{\rm ext}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)}) = \Big\{ \Xi\in\mathcal{F} \,\Big|\, \Xi = \Xi^{(k)} + \sum_{n>k}\bar{\Xi}^{(n)} \text{ with } N\bar{\Xi}^{(n)} = n\bar{\Xi}^{(n)} \text{ for } n>k \\ \text{and } \Xi^{(k)}\in\mathcal{H}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)}) , \\ \text{and } \mathcal{B}_\Sigma\Xi = 0 \Big\} . \end{aligned} \tag{G.34}$$

In other words, $\mathcal{H}_{\rm ext}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)})$ is the subspace of $\mathcal{F}$ which contains the $\mathcal{B}_\Sigma$-invariants whose lowest order according to the filtration N are elements of $\mathcal{H}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)})$. To perform this construction, we write explicitly the constraint of $\mathcal{B}_\Sigma$-invariance of Ξ:

$$\mathcal{B}_\Sigma\Xi = 0 \quad\Longleftrightarrow\quad \begin{cases} \mathcal{B}_\Sigma^{(0)}\Xi^{(k)} = 0 \\ \mathcal{B}_\Sigma^{(1)}\Xi^{(k)} + \mathcal{B}_\Sigma^{(0)}\bar{\Xi}^{(k+1)} = 0 \\ \mathcal{B}_\Sigma^{(2)}\Xi^{(k)} + \mathcal{B}_\Sigma^{(1)}\bar{\Xi}^{(k+1)} + \mathcal{B}_\Sigma^{(0)}\bar{\Xi}^{(k+2)} = 0 \\ \vdots \end{cases} \tag{G.35}$$

We readily see that the first equation is automatically satisfied since $\Xi^{(k)}\in\mathcal{H}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)})$, and that the other ones are really constraints since, for example, $\mathcal{B}_\Sigma^{(1)}\Xi^{(k)}$ is not automatically $\mathcal{B}_\Sigma^{(0)}$-trivial. Thus, step 2 consists in solving this system of equations (which is finite due to the decomposition (G.33) being finite), starting with the cohomologies found in step 1.

Step 3: We finally test the triviality of the elements found in step 2 by the following construction. Let us define the sum:

$$\mathcal{H}_{\rm ext}(\mathcal{F},\mathcal{B}_\Sigma) = \bigoplus_{k=0}^{8}\mathcal{H}_{\rm ext}(\mathcal{F}^{(k)},\mathcal{B}_\Sigma^{(0)}) , \tag{G.36}$$

as well as the subspace $\mathcal{I}(\mathcal{F},\mathcal{B}_\Sigma)$ containing the trivial $\mathcal{B}_\Sigma$-invariants:

$$\mathcal{I}(\mathcal{F},\mathcal{B}_\Sigma) = \Big\{ \Theta\in\mathcal{F} \,\Big|\, \mathcal{B}_\Sigma\Theta = 0 \text{ and } \exists\,\hat{\Theta} \text{ such that } \mathcal{B}_\Sigma\hat{\Theta} = \Theta \Big\} . \tag{G.37}$$

<u>Remark</u>: $\hat{\Theta}$ is *not* subject to the constraints defining $\mathcal{F}$, *i.e.* it is a general local functional in the fields with dimension 3, ghost number -1 and R-weight 0 such that $\mathcal{B}_\Sigma\hat{\Theta}$ belongs to $\mathcal{F}$.

Thus, the cohomology $\mathcal{H}(\mathcal{F},\mathcal{B}_\Sigma)$ is given by the quotient of $\mathcal{H}_{\rm ext}(\mathcal{F},\mathcal{B}_\Sigma)$ by $\mathcal{I}(\mathcal{F},\mathcal{B}_\Sigma)$:

$$\mathcal{H}(\mathcal{F},\mathcal{B}_\Sigma) = \mathcal{H}_{\rm ext}(\mathcal{F},\mathcal{B}_\Sigma)/\mathcal{I}(\mathcal{F},\mathcal{B}_\Sigma) . \tag{G.38}$$

Moreover, $\mathcal{I}(\mathcal{F},\mathcal{B}_\Sigma)$ can easily be calculated since it is simply given by the $\mathcal{B}_\Sigma$-variation of the general element displayed in G.1, equation (G.31).

We have now finished with the description of the general method and turn to the specific case of interest in this paper.

Step 1: Due to the constraints defining $\mathcal{F}$, the action of $\mathcal{B}_\Sigma^{(0)}$ on the elements of $\mathcal{F}$ reduces to:

$$\mathcal{B}_\Sigma^{(0)} = S_\mathcal{G} + S^{*(0)} + \int d^4x \left\{ +\hat{J}_\mu \frac{\delta}{\delta I_\mu} + \left(-i\nabla_\mu\varepsilon + \bar{\chi}\bar{\sigma}_\mu + \frac{1}{4}X_\mu\varepsilon - \frac{i}{8}X^\nu\varepsilon\sigma_{\mu\nu} \right) \frac{\delta}{\delta\Psi_\mu} \right.$$
$$\left. - \left(-i\nabla_\mu\bar{\varepsilon} + \chi\sigma_\mu - \frac{1}{4}X_\mu\bar{\varepsilon} + \frac{i}{8}X^\nu\bar{\varepsilon}\bar{\sigma}_{\mu\nu} \right) \frac{\delta}{\delta\bar{\Psi}_\mu} \right\} , \quad \text{(G.39)}$$

with X_μ defined in (2.2),

$$S_\mathcal{G} = \int d^4x \left\{ \nabla_\mu c^i \frac{\delta}{\delta A^i_\mu} - \frac{1}{2}f^{ijk}c^j c^k \frac{\delta}{\delta c^i} - f^{ijk}c^j \left(\hat{A}^{*k\mu}\frac{\delta}{\delta\hat{A}^{*i\mu}} + \lambda^k\frac{\delta}{\delta\lambda^i} + \lambda^{*k}\frac{\delta}{\delta\lambda^{*i}} \right.\right.$$
$$\left. -\bar{\lambda}^k\frac{\delta}{\delta\bar{\lambda}^i} - \bar{\lambda}^{*k}\frac{\delta}{\delta\bar{\lambda}^{*i}} + c^{*k}\frac{\delta}{\delta c^{*i}} \right) - T^i_{AB}c^i \left(\phi_B\frac{\delta}{\delta\phi_A} + \phi^*{}_B\frac{\delta}{\delta\phi^*{}_A} + \bar{\phi}_B\frac{\delta}{\delta\bar{\phi}_A} \right.$$
$$\left.\left. + \bar{\phi}^*{}_B\frac{\delta}{\delta\bar{\phi}^*{}_A} + \psi_B\frac{\delta}{\delta\psi_A} + \psi^*_B\frac{\delta}{\delta\psi^*_A} - \bar{\psi}_B\frac{\delta}{\delta\bar{\psi}_A} - \bar{\psi}^*{}_B\frac{\delta}{\delta\bar{\psi}^*{}_A} \right) \right\} , \quad \text{(G.40)}$$

corresponding to the $\mathcal{G}$-gauge BRS operator and

$$S^* = \int d^4x \left\{ \left(\sum_{\varphi\in\{A^i_\mu,\lambda^i,\phi_A,\psi_A\}} \frac{\delta\Sigma_{\rm inv}}{\delta\varphi}\frac{\delta}{\delta\varphi^*} \right) \right.$$
$$\left. + \left(\nabla_\mu\hat{A}^{*i\mu} + f^{ijk}(\lambda^{*j}\lambda^k + \bar{\lambda}^{*j}\bar{\lambda}^k) + T^i_{ab}(\phi^*_a\phi_b + \bar{\phi}^*_a\bar{\phi}_b - \psi^*_a\psi_b - \bar{\psi}^*_a\bar{\psi}_b) \right) \frac{\delta}{\delta c^{*i}} \right\},$$
$$S^* = \sum_{k=0}^4 S^{*(k)} , \qquad \text{with} \quad [N, S^{*(k)}] = kS^{*(k)} , \quad \text{(G.41)}$$

where $\Sigma_{\rm inv}$ is given by (4.12).

We see that $(I_\mu, \hat{J}_\mu)$ appear in $\mathcal{B}_\Sigma^{(0)}$ as a BRS-doublet, from what follows[11] that the $\mathcal{B}_\Sigma^{(0)}$-cohomology can be choosen independent of these fields.

The calculation of the cohomology spaces $\mathcal{H}(\mathcal{F}^{(k)}, \mathcal{B}_\Sigma^{(0)})$, $k = 0,\ldots,8$ is then standard: we write the most general element of $\mathcal{F}^{(k)}$ which is independent of $(I_\mu, \hat{J}_\mu)$, apply the condition of $\mathcal{B}_\Sigma^{(0)}$-invariance and finally test the triviality of the remaing candidate. We give here only $\mathcal{H}(\mathcal{F}^{(0)}, \mathcal{B}_\Sigma^{(0)})$ since, anticipating the Step 2, we found that the extensions of $\mathcal{H}(\mathcal{F}^{(k)}, \mathcal{B}_\Sigma^{(0)})$ for $k = 2,\ldots,8$ are all empty. Thus, the most general element $\Xi^{(0)}$ of $\mathcal{H}(\mathcal{F}^{(0)}, \mathcal{B}_\Sigma^{(0)})$ is

$$\Xi^{(0)} = \int d^4x \sqrt{g} \left(\alpha_1 F^{i\mu\nu}F^i_{\mu\nu} + \alpha_{2(AB)(CD)}\phi_A\phi_b\bar{\phi}_C\bar{\phi}_D + \alpha^i_{3AB}\phi_A(\bar{\psi}_B\bar{\lambda}^i) \right.$$
$$+ \bar{\alpha}^i_{3AB}\bar{\phi}_A(\psi_B\lambda^i) + \alpha_{4(AB)C}(\psi_A\psi_B)\phi_C + \bar{\alpha}_{4(AB)C}(\bar{\psi}_A\bar{\psi}_B)\bar{\phi}_C$$
$$\left. + \alpha_5 C^{\mu\nu\rho\lambda}C_{\mu\nu\rho\lambda} + \alpha_6\hat{G}^{\mu\nu}\hat{G}_{\mu\nu} \right) , \quad \text{(G.42)}$$

[11] see e.g. Section 5.2. of [6]

where $\alpha_1, \alpha_5, \alpha_6$ are arbitrary constants and $\alpha_2, \alpha_3, \alpha_4$ are $\mathcal{G}$-invariant tensors. The Weyl tensor $C_{\mu\nu\rho\lambda}$ is

$$C_{\mu\nu\rho\lambda} = R_{\mu\nu\rho\lambda} - \frac{1}{2}\left(g_{\mu\rho}R_{\nu\lambda} - g_{\nu\rho}R_{\mu\lambda} - g_{\mu\lambda}R_{\nu\rho} + g_{\nu\lambda}R_{\mu\rho}\right) + \frac{1}{6}R\left(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho}\right) \ , \tag{G.43}$$

where the Riemann tensor $R_{\mu\nu\rho\lambda}$, the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R are defined in C, equation (C.6).

Finally, the R-curvature tensor $\hat{G}_{\mu\nu}$ is

$$\hat{G}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu \ . \tag{G.44}$$

Steps 2 and 3: The extension of $\mathcal{H}(\mathcal{F}^{(k)}, \mathcal{B}_\Sigma^{(0)})$ is then straightforward (although going through tedious calculations) and proceeds as explained in solving (G.35). Finally, showing that the extended functionals are not trivial leads to the following most general element $\Sigma_{\rm ph}$ of $\mathcal{H}(\mathcal{F}, \mathcal{B}_\Sigma,)$:

$$\Sigma_{\rm ph} = Z_G \frac{\partial\Sigma}{\partial G} + Z_{(ABC)} \frac{\partial\Sigma}{\partial\Lambda_{(ABC)}} + \bar{Z}_{(ABC)} \frac{\partial\Sigma}{\partial\bar{\Lambda}_{(ABC)}} + \Sigma_{\rm CSG} \ ,$$

where $\Sigma_{\rm CSG}$ is the purely conformal supergravity action [21], which is of the form

$$\Sigma_{\rm CSG} = Z_{\rm CSG} \int d^4x \, \sqrt{g} \left(C^{\mu\nu\rho\lambda} C_{\mu\nu\rho\lambda} - \frac{3}{4} \hat{G}^{\mu\nu} \hat{G}_{\mu\nu} + \cdots \right) \ . \tag{G.45}$$

The dots involve the gravitational fields ${\rm e}_\mu{}^a, \Psi_\mu, \hat{B}_\mu$ and contain among other a kinetic part for the gravitino Ψ_μ.

The interpretation of these different counterterms is done in the text, just after (5.7).

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